Analysing Urban Areas using Multiple Track POL-InSAR Data at L-Band

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Abstract

This paper extends two single polarization multibaseline interferometric SAR high resolution techniques to the fully polarimetric configuration: the Music method and one approach based on maximum likelihood. These new algorithms enhance the height estimation of scatterers by calculating optimal polarization combinations and allow to determine their physical characteristics. First experimental results are shown estimating the height of buildings and their polarimetric features using multibaseline Pol-InSAR data acquired by DLR’s E-SAR system.

1 Introduction

Interferometric SAR (InSAR) is a technique to determine the height location of scatterers, whereas their physical properties can be extracted by SAR polarimetry. A first mathematical formulation to estimate the vertical location of scattering mechanisms using POL-InSAR data was introduced in [1]. Recently, polarimetric high resolution techniques were applied to polarimetric tomography [2]. This paper presents a new way of analysing polarimetric multibaseline (MB) InSAR data by adapting two high resolution (HR) algorithms to this configuration. In section 2, the conventional single polarization signal model, the Music algorithm and the ML approach for MB InSAR height estimation is outlined. Section 3 describes the generalization to the fully polarimetric MB InSAR constellation: First, the signal model is adapted to deal with a basis of three polarizations, then the Music and ML estimators are formulated in a rigorous mathematical way and their features are described. Finally, first experimental results are shown in section 4, applying these new methods to estimate the height of scatterers and their physical properties using fully polarimetric MB InSAR data of the city of Dresden acquired by DLR’s E-SAR system.

2 Height Estimation using Conventional HR Techniques

2.1 MB InSAR Signal Model

An MB InSAR signal model for conventional single polarization HR algorithms [3, 4] is as follows: The vector $x \in \mathbb{C}^p$ of the received signals with $p$ sensors and $q$ sources is expressed as

$$x = \sum_{i=1}^{q} a(z_i)s_i + n$$  (1)

where $n \in \mathbb{C}^p$ is the noise vector and $s_i$ is the signal of the $i$th source. The steering vector $a(z) \in \mathbb{C}^p$ has the form

$$a(z) = [a_1(z) \exp(jk_{z_1}z), \ldots, a_p(z) \exp(jk_{z_p}z)]^T$$  (2)

with the vertical wavenumber $k_{z_i} = \frac{4\pi}{\lambda \sin \theta(z_i)}$. The corresponding matrix notation is

$$x = A(z)s + n$$  (3)

with the matrix $A(z) \in \text{Mat}_{p,q} (\mathbb{C})$ of steering vectors

$$A(z) = [a(z_1), \ldots, a(z_q)]$$  (4)

the signal vector $s = [s_1, \ldots, s_q]^T \in \mathbb{C}^q$ and the height vector $z = [z_1, \ldots, z_q]^T \in \mathbb{R}^q$. The sample correlation matrix $\hat{R} \in \text{Mat}_{p,p}(\mathbb{C})$ is computed by

$$\hat{R} = \frac{1}{M} \sum_{i=1}^{M} x_i x_i^H$$  (5)

where $H$ denotes transpose, complex conjugate. The signals and noise are assumed to be uncorrelated. The noise is modeled as a complex Gaussian random vector with zero mean and covariance matrix $\sigma^2 I$.

2.2 MB InSAR Music Algorithm

Let $q$ denote the supposed number of dominant scatterers. Then the number of eigenvalues of the noise subspace equals $N = p - q$, and the matrix $E_N \in \text{Mat}_{p,N}(\mathbb{C})$ of the corresponding eigenvectors spans this subspace. The spectrum of the single polarization Music method [3] can be calculated by

$$P_{MU}^M(z) = \frac{1}{a^H(z)E_N E_N^H a(z)}$$  (6)

The subspace spanned by the noise eigenvectors must at least be of dimension one ($p \geq q + 1$).
2.3 MB InSAR ML Estimator

Wax et al. [4] proposed an ML estimator based on the Gaussian noise hypothesis which remains optimal even for correlated signals unlike Music. The projection matrix \( P_{A(z)} \in \mathbb{Mat}_{p,p}(C) \) onto the signal subspace is defined by

\[
P_{A(z)} = A(z) \left( A^H(z)A(z) \right)^{-1} A^H(z) \tag{7}
\]

and the orthogonal projection \( P_{A(z)}^\perp \in \mathbb{Mat}_{p,p}(C) \) onto the noise subspace is

\[
P_{A(z)}^\perp = I - P_{A(z)}. \tag{8}
\]

Expressed by means of the eigendecompositions of the matrices \( P_{A(z)} \) and \( P_{A(z)}^\perp \) with the nonzero eigenvalues \( l_i^S \geq \ldots \geq l_i^S \) and \( l_i^N \geq \ldots \geq l_i^N \), the ML height estimator is

\[
z_q = \arg \min_{z} \alpha(z, q) \tag{9}
\]

with

\[
\alpha(z, q) = \left( \prod_{i=1}^{q} l_i^S(z) \right) \left( \frac{1}{p - q} \sum_{i=1}^{p-q} l_i^N(z) \right)^{p-q}. \tag{10}
\]

A way to solve this nonlinear, multimodal q-dimensional minimization problem is a simulated annealing approach [6]. Furthermore, the number of sources can be estimated by the MDL principle [4].

3 Fully Polarimetric MB InSAR HR Techniques

In this section, the HR techniques are extended to the fully polarimetric MB InSAR configuration. The following adaptation to the fully polarimetric case not merely increases the number of observables, but especially finds the optimal polarimetric combination for height estimation.

3.1 Polarimetric MB InSAR Signal Model

The vector \( x \in C^p \) of the received signals with \( p \) sensors and \( q \) sources is modeled as

\[
x = \sum_{i=1}^{q} b(z_i, k_i)s_i + n \tag{11}
\]

where \( b(z_i, k_i) \in C^p \) is the MB polarimetric interferometric (MBPI) steering vector of the \( i \)th source. Let \( \gamma(l) \in C, l = 1, \ldots, 3 \), be distinct polarization states, e.g. the Pauli basis

\[
\begin{bmatrix} \gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)} \end{bmatrix}^T = \frac{1}{\sqrt{2}} [S_{HH+VV}, S_{HH-VV}, 2S_{HV}]^T. \tag{12}
\]

Then the steering vector associated to channel \( \gamma^{(l)} \) has the form

\[
a(z, \gamma^{(l)}) = [a_1(z, \gamma^{(l)}) \exp(jkz_s z), \ldots, a_p(z, \gamma^{(l)}) \exp(jkz_p z)]^T. \tag{13}
\]

The MBPI steering vector is the linear combination

\[
b(z, k) = k^{(1)} a(z, \gamma^{(1)}) + k^{(2)} a(z, \gamma^{(2)}) + k^{(3)} a(z, \gamma^{(3)}) \tag{14}
\]

with the fully polarimetric vector \( k \in C^3 \)

\[
k = \begin{bmatrix} k^{(1)}, k^{(2)}, k^{(3)} \end{bmatrix}^T \tag{15}
\]

that may be interpreted as a generalised scattering mechanism. This can be written in matrix notation

\[
b(z, k) = B(z)k \tag{16}
\]

with \( B(z) = [a(z, \gamma^{(1)}), a(z, \gamma^{(2)}), a(z, \gamma^{(3)})] \). The final polarimetric signal model for all \( q \) sources is

\[
x = C(z)Ks + n \tag{17}
\]

with the matrix \( C(z) = [B(z_1), \ldots, B(z_q)] \) and the block diagonal polarization matrix \( K \in \mathbb{Mat}_{3q}(C) \)

\[
K = \text{diag} [k_1, \ldots, k_q]. \tag{18}
\]

The correlation matrix is defined like in section 2.1, eq. (5).

3.2 Polarimetric MB InSAR Music Algorithm

If \( q \) is the hypothesised number of scatterers, the matrix of the noise eigenvectors is \( E_N \in \mathbb{Mat}_{p,N}(C) \) with \( N = p - q \). The spectrum of Music for the fully polarimetric SAR configuration is

\[
P_{FP, M}(z) = \frac{1}{\lambda_{\text{min}}(B^H(z)E_N E_N^H B(z))} \tag{19}
\]

with \( \lambda_{\text{min}} \) the smallest eigenvalue of the \( 3 \times 3 \) Hermitian linear system

\[
B^H(z)E_N E_N^H B k_{\text{min}} = \lambda_{\text{min}} k_{\text{min}}. \tag{20}
\]

The eigenvector \( k_{\text{min}} \) describes the physical features of the scatterer and permits a polarimetric analysis [5]. The linear system (20) must be of full rank, otherwise \( \lambda_{\text{min}} = 0 \) and the spectrum (19) is infinite and the height cannot be determined. A necessary criterion for the linear system having full rank is \( p \geq q + 3 \).

In the situation of diverse cross-polar components, four polarization states have to be included; the structure of the associated linear system is obvious. In this case, it is necessary that \( p \geq q + 4 \) for the matrix to be nonsingular. In a similar way, other HR techniques like the one proposed by Capon can be easily extended to the fully polarimetric configuration.
3.3 Polarimetric MB InSAR ML Estimator

Using the abbreviation $D = C(z)K$ for the matrix in eq. (17), and keeping in mind that the matrix $D$ depends both on height $z$ and polarization $K$, the projection matrix $P_D \in \text{Mat}_{p,p}(\mathbb{C})$ onto the signal subspace is

$$P_D = D (D^H D)^{-1} D^H$$

and the orthogonal projection $P_D^\perp \in \text{Mat}_{p,p}(\mathbb{C})$ onto the noise subspace is

$$P_D^\perp = I - P_D.$$  

(21)

Utilizing the decompositions of the matrices $P_D\hat{R}P_D$ and $P_D^\perp\hat{R}P_D^\perp$, the ML height estimator is

$$\hat{z}_q(K) = \arg \min_{(z,K)} \beta(z_q, K)$$

(23)

with

$$\beta(z_q, K) = \left( \prod_{i=1}^{q} t_i^S(z_q, K) \right) \left( \frac{1}{p-q} \sum_{i=1}^{p-q} t_i^N(z_q, K) \right)^{p-q}.$$  

(24)

The extension to the diverse cross-polarization situation is straightforward by incorporating four polarization states in the concept.

4 Experimental Results

To demonstrate the performance of the above introduced algorithms, some results using different configurations are presented next. In the following, the spectra were calculated in the range $[-10m, \frac{3}{4}H_{amb} - 10m]$, where $H_{amb}$ represents the height ambiguity. To allow a comparison between the Music and ML algorithms, the spectrum of the ML estimator is defined as $P^{ML} = \frac{1}{4}$ with $\beta$ defined by eq. (24). First, the spectrum of a sample pixel located in the layover area of a building was computed by the Music algorithm for a small baseline ($\approx 10m$) in both single polarization ($VV$) and fully polarimetric mode as depicted in Figure 1. With the assumption of a single dominant scatterer, there is no apparent difference between these two cases concerning the height estimation. However, the fully polarimetric Music method permits a polarimetric analysis by means of the scattering vector $k_{min}$ of the linear system (20). Three canonical scattering mechanisms are identified and assigned to the double bounce (DB), surface reflection (SR), and volume diffusion (VD) class, respectively [7]. While all these backscattering phenomena are present in the classification of the original data, the Music method is able to eliminate the volume diffusion class (Figure 2) suggesting that the algorithm is not biased by complex scenes including vegetation.

Figure 1: Small baseline ($\approx 10m$) single polarization and fully polarimetric Music spectra.

Figure 2: Identification of three basic scattering mechanisms (DB red, SR blue, VD green). Top: Original data, bottom: Polarimetric scattering mechanism generated by Music analysis.

To examine the ML estimator, another sample pixel associated to double bounce, wall ground interaction was selected. The small baseline single polarization combination exhibits two peaks of same power, whereas the dual polarimetric ML estimator ($HH +VV$ and $HH -VV$) is capable to attenuate noticeably the second maximum (Figure 3, top) facilitating considerably the height extraction. This effect is even more pronounced in the dual baseline ($\approx 40m$ for large baseline) case (Figure 3, bottom): For one polarization, there are several strong maxima whose powers are visibly suppressed by the dual polarimetric ML approach. A detailed investigation indicates that the spurious peaks are induced by height ambiguities. Finally, the height of buildings is estimated by the small baseline fully polarimetric Music method. Figure 4 shows the extracted height of two edifices lying on a sample line inside the black box of Figure 2 (top). Samples belonging to low correlated or shadowed areas are masked out by a criterion based on a threshold of the amplitudes and coherence. It has to be borne in mind that layover effects are still visible. The 3D image (Figure 5) shows two buildings colorcoded by the angle $\alpha_1$ describing the mechanism of the dominant scatterer: blue represents surface reflection ($\alpha_1 \approx 0$), red double bounce ($\alpha_1 \approx \pi/2$).
5 Conclusion

This paper has introduced two HR techniques in their most general form: they can be employed for fully polarimetric MB InSAR configurations in all possible combinations. They optimize the polarizations for height estimation of scatterers and permit to determine their physical behaviour. First experimental results suggest that the retrieval of building height and polarimetric properties of the scatterers is possible.

In the future, the thorough analysis of these new methods has to be continued, e.g. by examining the question which minimal configuration is best suited to estimate the height of buildings in order to reduce the necessary amount of data. In this respect, the estimation results already indicate that a dual polarimetric combination with $HH + VV$ and $HH - VV$ polarizations should be sufficient for yielding accurate results. These new algorithms will be also utilized in the field of polarimetric SAR tomography, e.g. to evaluate volumetric media height structure.

References