Complex scene analysis from Time-Frequency statistics of POLSAR data

Laurent Ferro-Famil
University of Rennes 1, IETR
Campus de Beaulieu, Bat 11D, 263 Avenue General Leclerc
F-35042 Rennes, France
Email: Laurent.ferro-Famil@univ-rennes1.fr

Andreas Reigber
Berlin University of Technology
Computer Vision and Remote Sensing Department
Franklinstr. 28/29 FR3-1, D-10587 Berlin, Germany.
Email: anderl@cs.tu-berlin.de

Abstract—This article presents a statistical approach for the study of POLSAR images using Time-Frequency (TF) correlation properties. POLSAR information is analyzed using a linear time-frequency (TF) decomposition which permits to describe a scene polarimetric behavior for different azimuth angles of observation and frequencies of illumination. A TF signal model is proposed and studied using two statistical descriptors related to the signal stationarity aspect and coherence in the time-frequency domain. These indicators are shown to provide complementary information for an enhanced description of the scene.

I. INTRODUCTION

Conventional scattering analysis and geophysical parameter retrieval techniques from strip-map synthetic aperture radar (SAR) data generally assume that scenes are observed in the direction perpendicular to the flight track and at a fixed frequency, equal to the emitted signal carrier frequency. These assumptions may lead to erroneous interpretations over complex targets, characterized by anisotropic geometrical structures, showing varying electromagnetic behavior as they are illuminated from different positions and at different frequency components during SAR integration. This paper presents a new approach for the study of POLSAR data, using a TF analysis based on second order statistics evaluated in the azimuth direction.

II. TF ANALYSIS OF SAR IMAGES

The time-frequency approach developed in this study is based on the use of a two-dimensional Short Time Fourier Transform (STFT). This kind of transformation permits to decompose of a two-dimensional signal, \( s(l) \), with \( l = [x, y] \), into different spectral components, using a convolution with an analyzing function \( g(l) \), as follows [1]:

\[
s(l_0; \omega_0) = \int s(l)g(l - l_0) \exp(j \omega_0(l - l_0)) \, dl \tag{1}
\]

where \( \omega = [\omega_x, \omega_y] \) indicates a position in frequency, and \( s(l_0; \omega_0) \) represents the decomposition result around the spatial and frequency locations \( l_0 \) and \( \omega_0 \). SAR images are built from measurements of a scene electromagnetic response over a large frequency band and a wide range of observation angles. In the case of already synthesized SLC SAR images, the spectral position, \( \omega = [\omega_{ry}, \omega_{az}] \) can be linked to the frequency of illumination, \( f \), and to the angle of observation in azimuth, \( \phi \), as :

\[
\omega_{ry} = \frac{4\pi}{c}(f - f_c), \quad \omega_{az} = \frac{4\pi}{c}f_c\sin(\phi), \tag{2}
\]

where \( f_c \) is the carrier frequency and \( c \) the light velocity. Variations of the backscattered signal with \( \omega \) may be interpreted as anisotropic or frequency selective behaviors of particular media or scatterers and linked to some of their physical properties [2]–[6]. This paper concentrates on the TF analysis of SAR data characteristics in the azimuth direction which provides the most useful information for building characterization.

III. TF ANALYSIS USING POLSAR DATA STATISTICS

A. Polarimetric continuous TF analysis

Buildings may show a non stationary TF behavior in azimuth with a sudden a large variation of both SPAN and polarimetric indicators. This anisotropic behavior is due to the highly directional patterns of coherent scattering mechanisms which may occur as the radar faces a large artificial structure, such as a building [7]. This particular effect can only be observed if the building orientation with respect to the radar flight track falls within the processed antenna azimuth aperture. On the contrary, oriented buildings and vegetated areas, like forest patches, show stationary behaviors. The identification of buildings from their TF response thus requires an additional criterion to complement the stationarity information. It is known that man-made objects are likely to have a coherent response, whereas natural media may be considered as random. The discrimination of such responses can be achieved by studying the coherence of the backscattered polarimetric signal in the Time-Frequency domain and requires the use of an adequate TF polarimetric SAR (PolSAR) signal model.

B. PolSAR data TF model

The proposed TF signal model is given by the following expression, where the spatial coordinates, \( I \), have been omitted :

\[
s(\omega) = t(\omega) + c(\omega) \tag{3}
\]

The signal \( s(\omega) \) contains the full coherent polarimetric polarization information and can be associated to a well-known
The likelihood terms in (7) are maximized by replacing the sub-spectra do not overlap and

\[ k(\omega) = \frac{1}{\sqrt{2}} [S_{hh}(\omega) + S_{ve}(\omega), S_{hh}(\omega) + 2S_{he}(\omega)]^T \]

(4)

where \( S_{pq}(\omega) \) represents an element of the \( 2 \times 2 \) scattering matrix \( S \) sampled at the frequency coordinates \( \omega \). The signal described in (3) is composed of two contributions:

- The term \( t(\omega) \) is highly coherent and can be associated to a deterministic or almost deterministic target response. Depending on the structure of the observed object, the response can be non-stationary.
- The second term, \( c(\omega) \), represents the response of distributed environments. It is uncorrelated, but may follow a non-stationary behavior in particular cases.

This composite model may be tested using \( s(\omega) \) second order statistics:

- The coherence of \( s(\omega) \) can be used to determine the dominant component within the pixel under consideration. A high value indicates that \( t(\omega) \) is the most important term in (3), and a low one corresponds to scattering from an incoherent, distributed, medium.
- The stability of the dominant component can then be tested by studying the stationarity of the variance of \( s(\omega) \).

C. TF second order statistics

A polarimetric TF target vector is built by gathering the PolSAR information sampled at \( R \) spectral coordinates \( k_{TF} = [k_T(\omega_1), \ldots, k_T(\omega_R)]^T \). The sampling coordinates, \( \omega_i \), and the frequency domain resolution of the analyzing function \( g \) are chosen so that the the \( R \) sub-spectra do not overlap and span the whole full resolution spectrum [4]. A polarimetric TF sample covariance matrix, \( T_{TF} \), is then computed as follows

\[ T_{TF-pol} = \langle k_{TF}k_{TF}^\dagger \rangle = \begin{bmatrix} T_{11} & \cdots & T_{1R} \\ \vdots & \ddots & \vdots \\ T_{R1} & \cdots & T_{RR} \end{bmatrix} \]

(5)

1) Indicator of stationary behavior: Stationarity is assessed by testing the fluctuations of the variance of the signal at the the different spectral locations. In the polarimetric case, the signal sample variance is given by a \( 3 \times 3 \) polarimetric coherency matrix, i.e. by the diagonal terms of the \( T_{TF-pol} \) matrix : \( \{T_{ii}\}_{i=1,R}. \) The polarimetric TF response is considered as stationary if the sample \( T_{ii} \) matrices, assumed to follow independent complex Wishart distributions \( T_{ii} \sim W_C(n_i, \Sigma_{ii}) \) with \( n_i \) looks, have the same expectation \( \Sigma \). The corresponding hypothesis is given by:

\[ H_0 : \Sigma_{11} = \cdots = \Sigma_{RR} = \Sigma \]

(6)

The corresponding Maximum Likelihood (ML) ratio is:

\[ \Lambda = \frac{\max_{\Lambda} \mathcal{L}(\Sigma, \ldots, \Sigma)}{\max_{\Sigma_{ii}} \mathcal{L}(\Sigma_{11}, \ldots, \Sigma_{RR})} \]

(7)

The likelihood terms in (7) are maximized by replacing the expectation matrices by the ML estimates and the hypothesis is tested using the resulting ML test [2], which takes the following form:

\[ \Lambda = \prod_{i=1}^{R} \frac{|T_{ii}|^{n_i}}{|T_{11}|^{n_1}} \]

(8)

where \( T_i = \sum_{i=1}^{R} n_i T_{ii} \) and \( n_r = \sum n_i \)

Fig.1 presents a log-image of the \( \Lambda \) parameter on the Dresden test site, obtained with 4 spectral coordinates in the azimuth direction over the Dresden test site. The \( \Lambda \) parameter reaches high values over natural areas indicating a stationary spectral behavior. Over buildings, \( R \) decreases, pointing out the invalidity of the stationary hypothesis over such objects. Highly anisotropic pixels, such as those corresponding to the wall-ground dihedral reflection or specular reflection from oriented roofs are clearly identified in Fig.1 due to their very low stationary aspect.

2) Indicator of coherent behavior: In [5], the eigenvalues of a single-polarization covariance matrix have been used to derive a coherency indicator. These eigenvalues carry information on the correlation structure, but are also sensitive to potential PolSAR fluctuations due to non-stationarity. Under the hypothesis of uncorrelated spectral responses, the off-diagonal terms of the TF covariance matrix verify:

\[ H_0 : \Sigma_{ij} = 0 \quad \forall i \neq j \]

(9)

The corresponding ML ratio is given by:

\[ \Theta = \frac{\max_{\Sigma} \mathcal{L}(\Sigma_{11}, \ldots, \Sigma_{RR})}{\max_{\Sigma_{TF}} \mathcal{L}(\Sigma_{TF})} = \frac{|T_{TF}|^{n_1}}{\prod_{i=1}^{R} |T_{ii}|^{n_i}} \]

(10)

Fig. 1. Pauli image of Dresden (top), ML log-ratio image (bottom)
This expression can be rewritten as $\Theta = \left| \tilde{T}_{TF} \right|^{-1}$, with:

$$\tilde{T}_{TF-Pol} = \begin{bmatrix} 1 & \Gamma_{12} & \ldots & \Gamma_{1R} \\ \Gamma_{12}^T & 1 & \ldots & \ast \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma_{1R}^T & \ldots & \ast & 1 \end{bmatrix}$$

(11)

where $\Gamma_{ij} = T_{ii}^{-1/2}T_{ij}T_{jj}^{-1/2}$. The normalized covariance matrix, $\tilde{T}_{TF-Pol}$ results from the whitening of the TF polarimetric covariance matrix by the separate polarimetric information at each frequency location. This representation is then insensitive to spectral polarimetric intensity variations and is characterized by its off-diagonal matrices $\Gamma_{ij}$ which can be viewed as an extension of the scalar normalized correlation coefficient to the polarimetric case. The ML ratio in (10) is a function of the eigenvalues of $\tilde{T}_{TF-Pol}$, which reflect the correlation structure: flat for decorrelated responses ($\tilde{T}_{TF-Pol} \rightarrow I_d$), heterogeneous for correlated ones. Taking into account $\tilde{T}_{TF-Pol}$ peculiar form, a correlation indicator, named TF-Pol coherence, can be defined as:

$$\rho_{TF-Pol} = 1 - \left| \tilde{T}_{TF-Pol} \right|^{-1}$$

(12)

Figure 2 presents an image of $\rho_{TF-Pol}$ over the Dresden test site, computed from 4 spectral locations in the azimuth direction. As expected, the TF-Pol coherence is high over buildings due to the presence of strong coherent reflectors. It can be also noticed that buildings are identified independently of their orientation.

IV. POLSAR TF CORRELATION PROPERTIES

A. PolSAR TF autocorrelation function

PolSAR TF correlation characteristics have been partially studied in section III by testing the presence of correlation structure between different frequency locations. The analysis of PolSAR TF correlation characteristics over pixels detected as coherent may provide more information on the nature of scattering mechanisms and require the use of TF autocorrelation function. The sample autocorrelation function of a stationary signal $s(t)$ is defined as:

$$\varphi_{ss}(\Delta t) = \langle s(t) \ast s(t + \Delta t) \rangle_t$$

(13)

where the averaging is performed over the whole domain of definition of $s(t)$. For non-stationary signals, i.e. signals whose statistical properties vary with $t$, this definition is generally inappropriate and is replaced by the local autocorrelation:

$$\varphi_{ss}(t_0, \Delta t) = \langle s(t_0 + t) \ast s(t_0 + t + \Delta t) \rangle_{t \in D_t}$$

(14)

where the correlation is computed around the time location $t_0$, within a restricted domain given by $D_t$. The autocorrelation of a TF signal $s(l; \omega)$ can be defined in several different ways, depending on the domain of study. In this paper, we concentrate on the analysis of the signal spectral correlation properties, and use the following autocorrelation expression:

$$\varphi_{ss}(l; \omega_0, \Delta \omega) = \langle s(l; \omega_0) \ast s(l; \omega_0 + \Delta \omega) \rangle_{l \in D_l}$$

(15)

where the sample mean is performed in the space domain, whereas the signal shift is realized in the frequency domain. This expression may be normalized by the signal energy around the frequency locations $\omega_0$ and $\omega_0 + \omega$:

$$\nu_{ss}(1; \omega_0, \Delta \omega) = \frac{\varphi_{ss}(1; \omega_0, \Delta \omega)}{\sqrt{\varphi_{ss}(1; \omega_0, 0) \varphi_{ss}(1; \omega_0 + \Delta \omega, 0)}}$$

(16)

where $0 \leq |\nu_{ss}| \leq 1$. The expression given in (16) can be generalized to the polarimetric case using:

$$\varphi_{ss}(w_{\omega_0}, w_{\omega_0 + \Delta \omega}; l) = w_{\omega_0}^l T(l; \omega_0, \Delta \omega) w_{\omega_0 + \Delta \omega} \ast w_{\omega_0 + \Delta \omega}$$

(17)

where $w_{\omega_0}$ and $w_{\omega_0 + \Delta \omega}$ represent 3-element projection vectors [9] at the analysis frequency locations and $T(l; \omega_0, \Delta \omega)$ is a TF $(3 \times 3)$ correlation matrix defined as:

$$T(l; \omega_0, \Delta \omega) = \langle s(l; \omega_0) \ast s(l; \omega_0 + \Delta \omega) \rangle$$

(18)

Figure 3 represents the mean and maximal normalized correlation values in the hh polarization channel, evaluated over a range of spectral locations $\omega_0$ for a given frequency lag $\Delta \omega$. The images in Fig.3 clearly indicate that even on pixels detected as coherent, the correlation information is not constant over the SAR acquisition, due to changes in both coherence level and type of scattering mechanism. The coherence optimization procedure described in [9] in the PolinSAR case may be applied to the autocorrelation function defined in (16):

$$|\nu_{opt_s}(1; \omega_0, \Delta \omega)| = \max_{w_{\omega_0}, w_{\omega_0 + \Delta \omega}} \nu(w_{\omega_0}, w_{\omega_0 + \Delta \omega}; l)$$

(19)

with $|\nu_{opt_s}| \geq |\nu_{opt}| \geq |\nu_{opt_{s'}}|$. Similarly to the PolinSAR case, a simple indicator may be built to assess the sensitivity of the optimal correlation information to polarization [10]:

$$0 \leq A = \frac{|\nu_{opt_s}| - |\nu_{opt_{s'}}|}{|\nu_{opt_s}|} \leq 1$$

(20)
scattering mechanisms, provided by the first eigenvector of the optimization procedure first described in [9]:

\[
\{ w_{\omega_0, \text{opt}}, w_{\omega_0 + \Delta \omega, \text{opt}} \} = \arg\max |\nu(w_{\omega_0}, w_{\omega_0 + \Delta \omega})| \quad (21)
\]

The projection vectors \( w_{\omega_0, \text{opt}} \) and \( w_{\omega_0 + \Delta \omega, \text{opt}} \) represent the scattering mechanisms at the analysis frequency locations which maximize the TF correlation. These quantities can be converted to scattering matrices \( S(\omega_0, \text{opt}) \) and \( S(\omega_0 + \Delta \omega, \text{opt}) \) from which a polarimetric distance can be computed [11]. Due to space limitation, images showing the cross- and co-pol nulls components of the polarimetric distance between \( S(\omega_0, \text{opt}) \) and \( S(\omega_0 + \Delta \omega, \text{opt}) \) over coherent pixels are not included in this paper. The difference between these distance metrics resides in the fact that cross-pol nulls variations can be compensated via a change of polarization basis, whereas co-pol nulls variations remain invariant under such a transformation. The generally high values of the cross-polarized distance and low co-pol nulls distances reveals that the optimal polarimetric scattering mechanisms at two different frequency locations significantly differ and correspond to changes of polarization basis which might be due to variations of the azimuth aspect angle.

REFERENCES


