ABSTRACT

The capability of DINSAR (Differential Interferometric SAR) for precise large-scale deformation analysis has been shown in various case studies. DINSAR has a high potential for monitoring deformation, but with a single image pair, only the velocity component parallel to the line-of-sight direction is measured. In order to retrieve the deformation velocity in both range and azimuth direction, an alternative technique for deformation analysis is the so-called spectral diversity approach. Spectral diversity is based on a phase comparison between different sub-aperture interferograms of the scene and can generally be regarded as a high-performance for estimating the mis-registration between complex SAR images.

In this paper, the following questions will be discussed based on spectral diversity and DINSAR results: 1) how to implement the spectral diversity technique for achieving the most accurate results; 2) how more reliable results can be obtained by a combination of DINSAR and spectral diversity technique; 3) how to extract the full 3D deformation vector from combination of ascending/descending passes and 4) how to extract a surface deformation map if the data sets are not perfectly coherent. Finally, a statistical analysis of every individual processing step and an error propagation analysis are undertaken. In order to make a quantitative analysis of the technique, ENVISAT data sets of the Bam earthquake in Iran are used.

1. INTRODUCTION

DINSAR (Differential Interferometric SAR) is well known technique to detect deformation with mm accuracy based on used radar wavelength. The main limitation of 3D (three-dimensional) deformation measurements made with interferometry is that interferogram gives only one direction - line of sight – (LOS) information. However, there is a growing investigation for mapping surface deformation in 3D. For this aim in the literature the amplitude offsets map related to mis-registration are used to extract deformation in azimuth direction. These techniques use cross correlation of the amplitude data of SAR images based on the coherence value maximization. Even, ascending and descending pair was used to obtain 3D deformation previous studies, the direct (without weighting) combination of different techniques with different accuracies causes an error at the final result of 3D deformation. The interpolation procedure also requires a large amount of computation in case of cross-correlation. In addition, if the phase unwrapping is needed, the error caused by phase unwrapping algorithm is also included in the last result.

The proposed -Spectral Diversity- technique for extraction of 3D deformation vector using complex SAR data is also based on finding mis-registration offsets caused by deformation. Spectral diversity technique was developed for the precise coregistration between two interferometric pairs and detection of residual motion errors in airborne SAR data [1] and [2]. While spectral diversity allows measuring a precise coregistration on SAR data, this technique can also be applied to estimate the local shifts occurred based on surface movement in range and azimuth directions between the interferometric pairs.

In this paper, two dimensional shift measurements based on spectral diversity technique of ascending and descending passes were combined with the aim of extraction of 3D deformation map. A weighted least square approach was used for homogenizing the measurements. In order to obtain general view of surface displacement, Delaunay triangulation and smooth interpolation were performed. Functional and stochastic relations allow the statistical analysis of every individual processing step and the error propagation analysis in the final results was investigated. Lastly, it is shown that the four interferometric pair is sufficient to infer a complete 3D surface displacement by the use of amplitude and phase information of interferometric SAR data.

2. DEFORMATION ANALYSIS USING DINSAR AND SPECTRAL DIVERSTY

2.1. Geometric Limits of DInSAR

The radar images include two kinds of information: one of them is brightness value based on target properties, the second is signal time delay which is based on the distances between target and sensor. If there is a second image for the same area, brightness and phase
information of images allow the interferogram generation by Hermitian multiplication of the images. This interferogram -phase differences – are wrapped around in cycles of 2π radians and must be unwrapped to obtain the absolute phase. Deformation on the surface between the two image passes also affects this phase value. This phase effect is included in the topographic phase value. The movement effects can be extracted if an external DEM (Digital Elevation Model) is suitable to be used to eliminate the topographic phase contribution. However, this phase based interferogram gives a L.O.S deformation information and it is not sensitive to detect the deformation which is perpendicular to the L.O.S direction (azimuth direction).

2.2. Spectral Diversity

The proposed -spectral diversity- technique for deformation analysis is described in literature [1], can be applied to interferometric pair after precise co-registration to find the range and azimuth phase shifts occurred based on deformation between the image passes. The basic idea of the use of spectral diversity is to measure mis-registration between the image amplitude by phase variation between sub-apertures. When there is time shift based on the surface movement, the spectral diversity works as shown in Fig.1.

In Fig.1, the technique is explained for only one direction, however, the procedure is same for azimuth and range direction and the technique can be applied separately through both direction. For example, in case of azimuth direction, each of two interferometric images IM and IS are split into a lower and upper azimuth band-pass filtered images, $I^L_M, I^U_M, I^L_S, I^U_S$. $I^U_M$ and $I^L_M$ indicate the upper bandwidth of master (M) and slave (S) images respectively and $I^U_S$ and $I^L_S$ indicate the lower bandwidth images. To keep the resolution as high as possible and prevent overlapping between bandwidths, the bandwidth of the bandpass filter is defined half of the azimuth bandwidth. Then, two interferograms are generated and the phase slope between a lower and upper bandpass is obtained as in the following:

$$\phi_1 = I^U_M * I^L_S$$
$$\phi_2 = I^L_M * I^U_S$$

$$d(a, r) = \phi_2 - \phi_1 \frac{2\pi}{\tau_{sr}}$$  \hspace{1cm} (1)

where * shows the complex conjugate and $\tau_{sr}$ indicates the pixel resolution (inversely proportional to processed bandwidth) in range or azimuth direction. Here, for ASAR IMS $\tau_r = 2x 4.052m$ and $\tau_a = 2x 7.803m$.

Additionally, before splitting the spectra, possible spectral weighting affects on the focused signal have to be removed. After sub-aperture extraction, weighting function should be applied again to all sub-aperture individually for minimising sidelobe effects. Despite the fact that the image resolution is slightly decreased, weighting with the aim of sidelobe reduction gives better result. In this study, a Kaiser window having a β value of 2.5 was used for weighting the sub-apertures. By the described approach, an accuracy of 0.01 sample can be obtained by spatial averaging (multilook) over a 50x50 resolution cells. It has to be emphasized that, in contrast to interferometric approaches, this technique can be performed both in range and azimuth, delivering deformation maps in both directions.

3. PRE-PROCESSING and GEOCODING

In order to make a precise co-registration and to eliminate the phase effect based on inaccurate orbit information, the DEM backward geocoding model with the help of SRTM (Shuttle Radar Topography Mission) was performed with the use of I.D.I.O.T. software [4]. Additionally, there is a wavenumber shift filtering procedure of two pair like in standard interferometric applications.

4. EXTRACTION OF 3D DEFORMATION VECTOR

Resolving 3D surface deformation should be done with care while deformation obtained from SAR data does not directly show the vertical deformation based on satellite passes which is not perpendicular to the surface. In the first situation (one capturing geometry), the phase information based on spectral diversity in range direction was evaluated giving 3D displacement vector on a pixel by pixel basis. With the aim of producing a projection from 3D displacement vectors $\vec{d} = (d_v, d_e, d_n)$ onto line of sight direction $\vec{d}_r$ and satellite heading direction $\vec{d}_a$, the following equations (2) and (3) were generated related to the fig. 2 where $\theta_{inc}$ is the incidence angle, $H$ is the heading angle (positive clockwise from the North) and $\vec{d} = (d_v, d_e, d_n)$ indicates deformations in vertical, north and east directions.

$$\vec{d}_r = \delta_r + d_v * \cos(\theta_{inc}) - \sin(\theta_{inc}) \sin(H) + d_e * \cos(H)$$  \hspace{1cm} (2)

$$\vec{d}_a = \delta_a + d_n * \cos(H) - d_v * \sin(H)$$  \hspace{1cm} (3)
Ground point DEM ($\lambda$, $\Phi$, $h$)

Ancillary and orbit parameters

Projection to SAR(row, column) reference

Master Scene

Range Filtering

Azimuth Filtering

Coregistration

Registered Slave Scene

Slave Scene

Filtering

Bandpass filtering and look extraction

Look1

Look2

Interferogram from first sublooks

Interferogram from second sublooks

Different Interferogram

Look1

Look2

Differenced Interferogram

Figure 1. The block diagram of spectral diversity processing

Pre-processing

worth the

Figure 2. Representation for rightlooking ascending satellite including relationship between interferometry phase observations in line of sight (LOS) $\vec{d}_{r}$ and in heading $\vec{d}_{a}$ direction and ground deformation $\vec{d} = (d_{r}, d_{a}, d_{s})$

In these equations $\delta_{r}$ and $\delta_{a}$ are measurements’ errors based on atmospheric noise, low coherence, poor knowledge about orbit geometry and DEM etc. In this study these two effects were neglected by using the pixels accomplishing the selection criteria (coherence threshold).

In the second situation, the range and azimuth changes are obtained from different viewing geometries by means of spectral diversity; one is on the ascending swath and the other is on the descending swath. Data from ascending and descending orbits have different look direction, providing linearly independent two LOS and heading measurements. Based on descending satellite geometry in equations (2) and (3) plus sign replaces minus sign and vice versa. For the descending geometry, equations are changed as in the following

$$\hat{d}_{r} = \delta_{r} + d_{r} \cos(\theta_{inc}) [d_{s} \sin(H) - d_{n} \cos(H)] \quad (4)$$

$$\hat{d}_{a} = \delta_{a} + d_{n} \cos(H) - d_{e} \sin(H) \quad (5)$$

Solving the three unknowns, spectral diversity technique was applied to both the ascending and descending pair. Radar spectral diversity method which was applied in range and azimuth direction gives two measurement results in each pixel; the slant range deformation $\vec{d}_{r}$ (with the same direction obtained from DINSAR) and the azimuth line deformation $\vec{d}_{a}$ (hardly observable via DINSAR). In order to translate these shifts into the ground displacements several corrections should be performed.

4. WEIGHTED LEAST SQUARE APPROACH (WLS)

The aim of the entire WLS model is to estimate $n$ unknowns using $m$ observables, with ($m \geq n$). This is done by searching a vector $\vec{d}$ in $\mathbb{R}^{n}$ such that $A\vec{d}$ is as close to $Q$ as possible ($Q = A\vec{d}$). If $A$ is an $mn \times n$ matrix with rank $n$, then $A^{T}A$ is non-singular and the linear system $Q = A\vec{d}$ has a unique least squares solution given by $\vec{d} = (A^{T}A)^{-1}A^{T}Q$. As it is often the case, the system $Q = A\vec{d}$ is inconsistent, so there is residual error of $\epsilon \in \mathbb{R}$ observations $Q$. Observations can be rewritten with $\epsilon$ which is guarantees that $A\vec{d} + \epsilon$ is as small as possible.

$$Q + \epsilon = A\vec{d}$$

$$\epsilon = A\vec{d} - Q \quad (6)$$

Minimize the residual error $\epsilon \epsilon^{T} \rightarrow \text{Min} \rightarrow 0$
If the columns of matrix A are linearly independent with the different level of accuracy, the both sides of the equation $e = Ad - Q$ can be multiplied by weight matrix P. Then, the optimal solution (6) for $\hat{d}$ including a weighting matrix P is rewritten as:

$$e^t P e = (Ad - Q)^t P (Ad - Q)$$
$$e^t P e = d^t A^t P Ad - 2Q^t P Ad - Q^t PQ$$

An expression for $d$ which minimises (7) can be derived as in the following

$$\frac{\partial (e^t P e)}{\partial d} = A^t P Ad - Ad PQ = 0$$

$$d = A^t PQ$$

By minimising of the residual error, with (8) the unknowns are found. The mean square error (MSE) of the calculation is

$$m^2_e = \frac{(e^t P e)}{m-n}$$

This equation shows the quantitative evaluation of the observations. Its value indicates how far the chosen equation model fits with the measured unknowns. After the calculations, significant deviations of $m^2_e$ from 1 indicate errors in the calculation.

The goal of using WLE here is to convert the estimated offsets ($Q$) in both direction into vertical, northing and easting components. According the equations (2), (3), (4) and (5); there are four observations for each pixel: two of them are mis-registration map in range direction and two of them are mis-registration map in azimuth direction. The vector of unknowns $\hat{d} \in \mathbb{R}$ for each pixel are component of deformations $\hat{d} = (d_r, d_n, d_e)$, that they are real and non-stochastic. The matrix A consists of the coefficients of deformation equations in three direction.

Although these linear equations seem solvable with simple linear equation calculation techniques, this is not necessarily true when the observations are obtained with different geometry and having variable phase accuracy. Weighting of observations is done by the observations' standard deviation values that is a widely used parameter quantifying the effect of interferometric phase noise. The standard deviation of phase is based on the number of samples and coherence value of interferometric pair. According to this, the standart deviation of phase shift information are calculated at each pixel through four images due to [1]. If the multilook is large enough, the standard deviation of phase shift, $\sigma_r$ and $\sigma_e$ are obtained with enough accuracy. Then diagonal weight matrix of observations P is defined as follows:

$$P = Q^{-1}_e \sigma_p C^{-1}_e$$

where $Q_e$ is the cofactor matrix of observations; $\sigma_p$ is the a priori variance factor that used for weight definition and $C_e$ is the variance matrix of observations.

In this way, observations with high accuracy (high coherence value = low standard deviation) get high weights and will therefore have a strong influence on the estimated unknowns and vice versa.

5. WORKING AREA

On 26 December 2003, city of Bam – south-eastern part of Iran - suffered from a strong earthquake (Mw=6.5). The earthquake caused a great damage in the urban centre (Fig. 3). The effect of the earthquake is already visible in the coherence map where the city centre appears decorrelated due to the earthquake; while normally urban areas show a very high correlation. In addition, the fault line can easily be detected from the coherence map.

6. EXPERIMENTAL RESULTS

This section tries to present and discuss the detection of deformation based on DINSAR and spectral diversity methods from co-seismic interferometric data sets. For this purpose, 3 descending orbits and 3 ascending orbits have been processed (see table 1). In fig. 4a, a descending co-seismic interferogram, depicting the slant range displacement due to the earthquake is shown, generated from the 03 December 2003 (before earthquake) and 07 January 2004 (after earthquake) ASAR products.
Because of the large displacement, the deformation pattern appears wrapped and its gradient could not be measured directly by DInSAR due to phase unwrapping and noise problems in the area close to the fault. In this area, the extrapolation of fringes or similar techniques have to be considered. From the same data pair, spectral diversity deformation maps in azimuth and range direction, respectively, were derived, shown in Fig. 4 (b and c). In contrast to Fig. 4a, the fault line, which is located in the north-south direction, can now easily be detected without any priori knowledge.

In order to make a quantitative deformation analysis, 6 profiles marked as black lines in Fig. 4(b) were analysed. As a result of the earthquake, surface movement of various amounts could be found in these profile analysis (see Fig. 5-6). According to Fig. 5, a discontinuity and a sign change at the fault crossing can be found in azimuth profiles. An analysis of the azimuth displacements also shows that the amount of deformation decreases from north to south direction. In the first horizontal profile, which is close to the city centre, the estimated azimuth deformation changes about 70 cm from the beginning of the profile to the end of the profile. In the last azimuth profile, this change is only between -0.01 cm to -0.22 cm (21 cm along the profile line). However, the SNR (signal to noise ratio) in range is much lower than the one in azimuth.

In addition to this “left-to-right” profile analysis, a “south-north” profile analysis was performed on the azimuth deformation map, shown in Fig. 6. This figure supports the assumption that both sides of the fault move away from each other: one of them tends more towards to north direction, the other more towards south direction. According to these deformation maps, profiles of displacement in azimuth and range change from -80 cm to 60 cm.

Figure 4. (a) Differential interferogram of earthquake area where 1 cycle indicates the 28mm deformation in LOS direction, (b) Azimuth direction spectral diversity deformation map where black arrow shows the plate movement of the both side of the fault line and (c) Range direction deformation map. In (b) and (c), colours denote the displacement amplitude in meters and these maps’ reduced spatial resolution is based on bandpass filtering in spectral diversity approach.

Figure 5. Profiles of azimuth shift map in left-right direction in meters.
Fig. 6. Profile analysis from azimuth shift phase map in the south-north direction.

After producing deformation maps and their profile files from ascending and descending pairs, these maps were combined by a weighted least square approach to deduce the full 3D deformation field of the Bam earthquake. Then, horizontal (north-east surface) and vertical sub-sampled displacement maps were produced, based on Delaunay triangulation and smooth interpolation. Fig. 7(a) shows vertical deformation, derived using the proposed approach, which fit very well with DInSAR results. In this figure it can be seen how the vertical deformation changes its sign through the fault line.

Figure 7. (a) Vertical $\vec{d}_v$ and (b) horizontal $\sqrt{\vec{d}_r^2 + \vec{d}_n^2}$ components of surface displacement obtained after solving the functions that they show the relationship between phase shift map and deformation

Figure 8. Horizontal displacement vectors that show the dominant directions of surface movement. Arrows indicates the magnitude $\sqrt{\vec{d}_r^2 + \vec{d}_n^2}$ of horizontal deformation and its direction on the north-east surface

Although, interferometry gives more precise results, the capability of obtaining deformation information in both range and azimuth direction makes spectral diversity technique strong and preferable in deformation analysis. In this application, the maximum standard deviation of azimuth shift is 0.008 pixel ($0.008 \times 4.05 \times 2 = \pm 6$ cm) and that of range shift is 0.01 pixel ($0.01 \times 7.80 \times 2 = \pm 16$ cm), where 4.05m and 7.80m indicate azimuth and range resolution respectively. These accuracy values are high enough for detecting strong deformation like earthquake. However these satisfactory results, the accuracy decreases in the low coherence areas, which are close to the fault and the city centre.

The error sources affecting deformation vector calculation can be classified in two classes; the single point error (un-correlated observations) and multi-temporal/spatial data set errors. The analysis of the single point accuracy was performed using root mean square error in three directions (see Fig. 9 and table 2). Multi-data set error is evaluated based on least-square definition, if the data set are complement each other there is less residual error and vice versa (see also Table 1). The used data and their spatial and temporal properties are shown in table 1.
<table>
<thead>
<tr>
<th>Data Set</th>
<th>Acquisition dates</th>
<th>Standard Deviation ($\sigma_0$)</th>
<th>Baseline (m)</th>
<th>Temporal Differences (day)</th>
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<tr>
<td>Descending</td>
<td>Ascending</td>
<td></td>
<td></td>
<td></td>
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<td>03.12.2003 --&gt; 11.02.2004</td>
<td>16.11.2003 --&gt; 29.02.2004</td>
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<td>$0.6$ - $2$</td>
<td>$70$ – $105$</td>
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<td>$570$ - $2$</td>
<td>$35$ – $105$</td>
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<td>$0.1842321$</td>
<td>$0.6$ – $30$</td>
<td>$70$ – $70$</td>
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</tbody>
</table>

Table 1 The co seismic interferometric pairs and their properties used in this study.

From point position accuracy analysis it becomes clear that the accuracy of the east component is generally lower than the one of the north component. This is supported by the derived error ellipses. If the error is randomly distributed, the ellipse shape should be close to circular. In Table 2, the calculated error ellipses for randomly selected pixels are shown for visualizing the results. This amount of error in east direction is expected based on SAR capturing geometry.

<table>
<thead>
<tr>
<th>Point Position (PP)</th>
<th>Point position error (cm)</th>
<th>Parameters of Error Ellipse</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td>Rotation (degree)</td>
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</table>

Table 2 The parameters of error ellipse where point position accuracy is $m_p = \sqrt{m_{x_p}^2 + m_{y_p}^2 + m_{z_p}^2}$

9. CONCLUSIONS

A comparison of spectral diversity and interferometry techniques for detecting of ground deformation has been carried out. Two-dimensional offset measurements based on spectral diversity technique from two complementary ascending and descending passes were combined with the aim of an extraction of a fully 3D deformation map. In order to combine the dataset, a weighted least square approach was used for homogenizing the measurement. In order to obtain a general view of surface deformation map Delaunay triangulation and smooth interpolation was performed.

It is demonstrated that the spectral diversity technique cannot only be used for precise coregistration, but also for resolving 3D movement vectors in vertical north-south and east-west directions. It is also demonstrated that 3D deformation map can be extracted from only 2 interferometric pairs, which is not possible by means of DInSAR. With the spectral diversity technique, only decimetre accuracy is possible, in contrast to millimetre accuracy in case of DInSAR. However, at least for analysing large co-seismic motion, this seems to be sufficient. Additionally, it is shown, that already with the relatively low spatial resolution of ENVISAT, the achieved deformation accuracy is sufficient for 3D deformation analysis. Recent development of SAR technologies will supply much higher spatial resolution in near future, which will further enhance the precision of the proposed approach.

References

