INTRODUCTION

Phase unwrapping in SAR interferometry is a key step to construct a topographic map from a SAR dataset. Several phase unwrapping methods have been developed. They generally use strong mathematical foundations but also need quality maps in order to estimate the reliability of the phase information and then improve the unwrapping process.

By construction, SAR imaging is a projection of a volume response onto a plane. Phase artefacts may appear during this projection. It is thus necessary to discriminate artefacts arising over man-made or volumetric targets, like urban area or forest, from those related to the SAR device, like noise.

Polarimetric interferometric SAR data analysis is a helpful tool to discriminate the different kinds of artefacts. Most of them are encountered over volume areas, under the form of a phase centre bias. Recently, many studies have been proposed to estimate the interferometric phase over forest areas (Papathanassiou and Cloude (1), Yamada et al. (2)). One of them is based on the ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) algorithm, often employed for Direction-Of-Arrival estimation using antenna arrays, proposed by Yamada et al. (2). For volume areas, this algorithm can directly retrieve the interferometric phases of the ground and the canopy.

Man-made targets are constituted of different kinds of scatterer and the interferometric phase estimation becomes complex.

The second kind of artefacts, related to the SAR device, can be resolved using interferometric speckle filters, Lee et al. (3).

This paper addresses a polarimetric approach to improve the interferometric phase estimation based on an extension of the ESPRIT algorithm applied to different kinds of scatterer. In a first part are presented the basics of SAR interferometry. The second part introduces the ESPRIT algorithm using polarimetric data with a description of the scattering model, the ESPRIT algorithm and the method to determine the number of contributions. In the last section results obtained using fully polarimetric SAR images of the DLR E-SAR airborne sensor in L-band repeat-pass mode are discussed.

SAR INTERFEROMETRY

SAR interferometry is an established technique to extract topographic information of a terrain. It is based on the generation of an interferogram between two complex SAR images \( S_1 \) and \( S_2 \) acquired from two slightly different incidence angles; the sensors \( A_1 \) and \( A_2 \) are separated by a baseline \( B \). A target, \( P \), is located on the scene at a height \( h \). \( R_1 \) and \( R_2 \) respectively represent the distance between sensors and target (Figure 1).

The complex interferogram, \( D = \Im \{ S_1 S_2^* \} \), is obtained by multiplying the first signal by the complex conjugate of the second:

\[
\tilde{D} = \arg(S_1 S_2^*) = \frac{4\tilde{D}}{\partial R} \bigg|_{R=R_1} = \frac{4\tilde{D}}{\partial R} \left( R_1, R_2 \right)
\]

with \( \tilde{D} = \tilde{D} + \tilde{D} \).

![Figure 1: Geometry of an interferometric measurement.](image-url)
The measured phase is a wrapped representation of the absolute phase difference.

PHASE ESTIMATION

Backscattered waves can be considered as the sum of different contributions corresponding to different scattering mechanisms. Depending on the nature of the observed medium, the result of this sum may corrupt the value of the interferometric phase. The use of the ESPRIT technique permits to separate the different scattering mechanisms and to estimate the main interferometric phase.

Data model

Using the geometric configuration of Figure 1, signals, $S_1$ and $S_2$, received by sensors $A_1$ and $A_2$ may be written as:

$$
S_i^m = \sum_{m=1}^{d} \sum_{l=1}^{d} \sum_{kl} d_{mk}^l e_{mk}^l + n_i^m
$$  \hspace{1cm} (2)

$$
S_i^m = \sum_{m=1}^{d} \sum_{l=1}^{d} \sum_{kl} d_{mk}^l e_{mk}^l + n_i^m
$$  \hspace{1cm} (3)

Where $kl$ denotes polarization channels (HH, HV, VV, VH), $\sum_{m=1}^{d} \sum_{l=1}^{d}$ denote the polarization state of the $m$-th local scatterer in $kl$ polarization. $\sum_{m=1}^{d} \sum_{l=1}^{d}$ denote the intensity of the $m$-th local scatterer. $R$ is the slant range distance from the master orbit. $\sum_{m=1}^{d} \sum_{l=1}^{d}$ is the range difference of the $m$-th scatterer between master and slave tracks. $n_i^m$ denotes additive Gaussian noise in the $kl$ polarization. Using matrix and vector notation, equations (2) and (3) may be written as:

$$
S_1 = A \| n_1
$$  \hspace{1cm} (4)

$$
S_2 = A \| n_2
$$  \hspace{1cm} (5)

The polarization state and the intensity of each local scatterer for both interferometric acquisitions are assumed to be almost identical:

$$
\sum_{m=1}^{d} \sum_{l=1}^{d} \sum_{kl} d_{mk}^l e_{mk}^l
$$  \hspace{1cm} (6)

Then, $S_2$ may be simplified as follows:

$$
S_2 = A \| e_n
$$  \hspace{1cm} (7)

with $\| = \text{diag}\{e_1, e_2, \ldots, e_d\}$

Equations (4) and (7) have the same form as those in the TLS-ESPRIT algorithm. Thus, the interferometric phase of each scatterer can be estimated from $\|$. 

ESPRIT algorithm

The TLS-ESPRIT algorithm is based on a covariance formulation. The different scattering coefficient are gathered into a single matrix representation under the form of a covariance matrix, $R_m$, defined as:

$$
R_m = \left\{k^T \right\}
$$  \hspace{1cm} (9)

with $k = [S_{nm}^{\text{HH}}, \sqrt{S_{nm}^{\text{HV}}}, S_{nm}^{\text{VV}}, \sqrt{S_{nm}^{\text{VV}}}, S_{nm}^{\text{VH}}]^T$

A $6 \times 6$ matrix is obtained by this formulation. Then, an eigendecomposition is computed:

$$
\langle R_m \rangle = EE^T = \sum_{i=1}^{6} \sum_{i} \sum_{i} R_m
$$  \hspace{1cm} (11)

The eigenvalues of $E$ can be decomposed into two matrices $E_x$ as follow:

$$
E_x = \left[ \sqrt{\|}, e_1, \ldots, \sqrt{\|}, e_d \right] \| \ E_x
$$  \hspace{1cm} (12)

Applying an eigendecomposition leads to:

$$
E_x \| E_x^T = E_y \| E_y
$$  \hspace{1cm} (13)

$E$ is partitioned into $d \| d$ submatrices:

$$
E = \left[ \begin{array}{cc}
E_{11} & E_{12} \\
E_{21} & E_{22}
\end{array} \right]
$$  \hspace{1cm} (14)

The eigenvalues of $\| = \| E_{11}, E_{22} \|$ are calculated. Roy and Kailath (4) show that the eigenvalues $\|$, correspond to the diagonal elements of $\|$. Then the interferometric phase of each local scatterer, $\|$, can be estimated by:

$$
\| = \text{arg}(\|)
$$  \hspace{1cm} (15)
Determination of the local scatterer number

Equation (11) gives 6 eigenvalues \( \lambda_1 \geq \ldots \geq \lambda_d > \lambda_{d+1} \geq \ldots \geq \lambda_6 \). The number of local contributions may be estimated using a criterion such as AIC or MDL, but due to the small number of observables, another method, based on the spectral analysis of the 3 dominant eigenvalues is preferred, Cloude and Pottier (5).

The magnitude distribution of the eigendecomposition of equation (11) is expressed by:

\[
\lambda_1 \geq \ldots \geq \lambda_d > \lambda_{d+1} > \ldots > \lambda_6 = \sigma_n^2
\]  

(16)

where \( \sigma_n^2 \) is the average noise power. \( d \) is the number of contributions. The estimation of \( d \) is obtained by using only the 3 dominant eigenvalues, \( \lambda_1, \lambda_2 \) and \( \lambda_3 \). The relative power associated to a contribution is calculated as the ratio of the concerned eigenvalue to the sum of the 3 eigenvalues:

\[
p_m = \frac{\lambda_m}{\lambda_1 + \lambda_2 + \lambda_3}, \quad \sum_{m=1}^{3} p_m = 1
\]  

(17)

The 3 relative eigenvalue spectrum may be described with 2 indicators, \( H \) and \( A \).

The entropy, \( H \), indicates the degree of randomness of the relative contribution spectrum and is defined as follows:

\[
H = - \sum_{m=1}^{3} p_m \ln(p_m), \quad 0 \leq H \leq 1
\]  

(18)

The anisotropy, \( A \), represents the relative importance of the secondary contributions and is calculated by:

\[
A = \frac{p_2 p_3}{p_1 (p_2 + p_3)}, \quad 0 \leq A \leq 1
\]  

(19)

Different combinations of \( H \) and \( A \): \( HA, H(1-A), (1-H)A \) and \( (1-A)(1-H) \) are used in order to estimate the number of contributions \( d \). The sum of the different contributions is equal to 1. In this case, determining the most important contribution to the constrained sum performs the estimation of the configuration for a given pixel:

- If \( 1-H \) is maximum, there is one preponderant contribution and a secondary one: \( d = 1 \) (Figure 2 (b)).

- If \( HA \) is maximum, the phenomenon has two equivalent contributions: \( d = 2 \) (Figure 2 (c)).

- If \( 1-A \) is maximum, then the backscattering phenomenon is random, in this case \( d \) is taken equal at 2 as shown in Figure 2 (d).

![Figure 2: Representation of the normalized contribution spectrum for different configurations.](image)

APPLICATION TO POL-IN-SAR DATA

The analysis method mentioned above is applied to experimental polarimetric SAR images data of the Oberpfaffenhofen test site, acquired at L-band, in repeat-pass mode, by the DLR E-SAR sensor.

The HH, HV, VV data sets are used for polarimetric phase estimation. Figure 3 shows an optical image of the Oberpfaffenhofen test site and Figure 4 shows the corresponding colour coded polarimetric.
Figure 3: Optical image of the Oberpfaffenhofen test site.

Figure 4: Colour coded polarimetric image.

Contribution estimation

Figure 5, Figure 6, and Figure 7 show the entropy, $H$, the anisotropy, $A$, and the result of the estimation of the number of contribution, $d$, using the entropy/anisotropy analysis presented in the last section, respectively.

Interferometric phase estimation

Using the local scatterer number given by this analysis, the interferometric phase $(\Delta_{\text{HH}}, \Delta_{\text{VV}})$ of each local scatterer is calculated by ESPRIT algorithm using equation (15). On Figure 8 and Figure 9 are plotted interferometric phases $(\Delta_{\text{HH}}, \Delta_{\text{VV}}, \Delta_{\text{HV}}$ and $\Delta_{\text{VH}})$ corresponding to profiles 1 and 2.

The profile 2 corresponds to a surface area. In this case, only ground scattering occurs and the scattering centre for all polarizations is located on the ground. Thus, interferometric phases obtained from the polarisation channels, HH and VV, or by ESPRIT algorithm are similar.
The profile 1 corresponds to a more complex area, mixing forest, bushes, surfaces, and buildings. The interferometric phase of each contribution is optimised using ESPRIT algorithm in contrary to using only HH or VV channel. These channels are affected by significant volume and ground scattering contribution. Over the forest area, ESPRIT algorithm leads two optimum interferometric phases. One corresponds to the ground contribution and the second to the canopy contribution. The result is even better over bushes part. Variations of the ground contribution are due to varying underlying topography but also to a low interferometric coherence involved by decorrelation in the overlying random volume. This has for effect to increase the interferometric phase centre. Over building part, there are also two contributions and corresponding two interferometric phases. One contribution corresponds to a double reflection between the ground and the wall, which is superposed to a simple reflection on the top of the building. For this reason a gap may be observed between interferometric phases obtained over building area.

Errors are due to low interferometric coherence but also to electromagnetic shadow in the hidden part of buildings.

CONCLUSION

In this paper a polarimetric approach for interferometric phase estimation based on the TLS-ESPRIT algorithm is presented. The technique detects the local interferometric phase of different contributions separately. These different contributions are obtained using an entropy/anisotropy analysis based method. These properties are demonstrated using fully polarimetric interferometric DLR, E-SAR data.

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REFERENCES