Multibaseline POL-InSAR Analysis of Urban Scenes for 3D Modeling and Physical Feature Retrieval at L-Band

Stefan Sauer*, Laurent Ferro-Famil*, Andreas Reigber† and Eric Pottier*

* University of Rennes 1, IETR Laboratory, SAPHIR Team, Bat. 11D, 263 Avenue General Leclerc, CS 74205, 35042 Rennes Cedex, France
Tel./Fax: ++33-223-235019 / 236963
Email: stefan.sauer@univ-rennes1.fr
† Berlin University of Technology, Computer Vision and Remote Sensing Group, Franklinstrasse 28/29, FR3-1, D-10587 Berlin, Germany
Tel./Fax: ++49-30-314-23276 / 21104

Abstract—This paper generalizes a multibaseline interferometric SAR model taking polarization diversity into account. Based on this formulation, two high-performance spectral analysis techniques are extended to the multibaseline POL-InSAR configuration. These new algorithms enhance the height estimation of scatterers by calculating optimal polarization combinations and allow to determine their physical characteristics. Applying the methods to urban scenes, experimental results show the retrieval of building height and polarimetric properties by means of single-baseline polarimetric datasets. Dual-baseline observations permit the solution of the layover problem by separating two contributions within one resolution cell. The algorithms are tested using multibaseline POL-InSAR data acquired by DLR’s E-SAR system over Dresden city.

I. INTRODUCTION

Interferometric SAR (InSAR) is a technique to determine the height location of scatterers, whereas their physical properties can be extracted by SAR polarimetry. A first mathematical formulation to estimate the vertical location of scattering mechanisms using POL-InSAR data was introduced in [1]. Recently, polarimetric spectral analysis techniques were applied to multibaseline (MB) Pol-InSAR data of urban areas [2], [3], [4]. This paper presents a new way of analyzing polarimetric multibaseline InSAR observations by adapting two spectral analysis techniques to this configuration. In section II, the conventional single polarization signal model, the MUSIC algorithm and an ML estimator for MB InSAR height estimation are outlined. Section III describes the generalization to the fully polarimetric MB InSAR set-up: The signal model is adapted to deal with four polarization channels and subsequently the MUSIC and ML estimators are formulated in a rigorous mathematical way and their features are described. Finally, experimental results are shown in section IV, applying these new methods to estimate the height of scatterers and their physical properties using fully polarimetric MB InSAR data of Dresden city acquired by DLR’s E-SAR system.

II. MULTIBASELINE INSAR SPECTRAL ANALYSIS ALGORITHMS

This section describes a single polarization multibaseline InSAR signal model and two MB InSAR spectral analysis techniques: the MUSIC algorithm and an ML estimator.

A. Multibaseline InSAR Signal Model

The single polarization multibaseline InSAR received signal with \( p \) sensors can be modeled as

\[
y(n) = t + c(n) + v(n)
\]

\[
= \sum_{i=1}^{N_t} \sqrt{\tau_i} \exp(j\psi_i) a(z_{ti})
\]

\[
+ \sum_{j=1}^{N_c} \sqrt{\sigma_j} x_j(n) \otimes a(z_{cj}) + v(n)
\]

with \( n = 1, \ldots, N \), the number of looks \( N \), and the Schur-Hadamard product \( \otimes \) (elementwise multiplication). The MB InSAR received data vector \( y(n) \in \mathbb{C}^p \) is assumed to be a Gaussian random process with nonzero mean and covariance matrix \( \mathbf{R} \in \mathbb{C}^{p \times p} \), i.e. \( y(n) \sim \mathcal{N}_C(\mu, \mathbf{R}) \). The first term, \( t \), is highly coherent and can be associated to a deterministic or almost deterministic target [5]. The second contribution, \( c(n) \), represents the response of distributed environments including the SAR speckle effect as multiplicative noise [6]. The additive white Gaussian noise \( v(n) \in \mathbb{C}^p \) has zero mean and power \( \sigma_v^2 \), i.e. \( v(n) \sim \mathcal{N}_C(0, \sigma_v^2 I) \). The number of backscattering sources \( N_s \), and \( N_{sc} \) of the coherent and incoherent component, respectively, are assumed to be known. The total number of scatterers is \( N_s = N_{sc} + N_{s_c} \). The reflectivity \( \tau \) and the height \( z \) of the scatterers as well as the complex argument \( \psi \) are considered to be deterministic unknown quantities. The steering vector \( a(z) \in \mathbb{C}^p \) for a general acquisition geometry is represented as

\[
a(z) = [1, \exp(jkz_2z), \ldots, \exp(jkz_pz)]^T
\]
with the vertical wavenumber \( \kappa_z = \frac{4\pi}{\lambda} \sin \theta \). The multiplicative noise \( x_i(n) \in \mathbb{C}^p \) is a Gaussian random vector with zero mean and covariance matrix \( C_i = E(x_i(n)x_i^H(n)) \) where \( H \) denotes transpose, complex conjugate. This model does not take multipath effects into account. The sample covariance matrix \( \hat{R} \in \mathbb{C}^{p \times p} \) is computed by

\[
\hat{R} = \frac{1}{N} \sum_{n=1}^{N} y(n)y^H(n). \tag{3}
\]

**B. Multibaseline InSAR MUSIC Algorithm**

Let \( N_s \) denote the supposed total number of scatterers. Then the number of eigenvalues of the noise subspace equals \( q = p - N_s \), and the matrix \( E_q \in \mathbb{C}^{p \times q} \) of the corresponding eigenvectors spans this subspace. The spectrum of the single polarization MUSIC method [7] can be calculated by

\[
\hat{P}_{MU}^{SP}(\gamma) = \frac{1}{a^H(z)E_qE_q^H} a(z). \tag{4}
\]

The spectrum peak locations indicate the height of the scatterers. The subspace spanned by the noise eigenvectors must be at least of dimension one (\( p \geq N_s + 1 \)).

**C. Multibaseline InSAR ML Estimator**

Wax [8] proposed an ML estimator based on the stochastic signals model which, unlike MUSIC [9], remains optimal even for correlated signals. Let \( A(z) \in \mathbb{C}^{p \times N_s} \) be the steering vector matrix of the \( N_s \) sources

\[
A(z) = [a(z_1), \ldots, a(z_{N_s})] \tag{5}
\]

with the height vector \( z = [z_1, \ldots, z_{N_s}]^T \in \mathbb{R}^{N_s} \). The projection matrix \( P_{A(z)} \in \mathbb{C}^{p \times p} \) onto the signal subspace is defined by

\[
P_{A(z)} = A(z)(A^H(z)A(z))^{-1}A^H(z) \tag{6}
\]

and the orthogonal projection \( P_{\perp A(z)} \in \mathbb{C}^{p \times p} \) onto the noise subspace is

\[
P_{\perp A(z)} = I - P_{A(z)}. \tag{7}
\]

Using the eigenvalues \( \lambda_1^S \geq \ldots \geq \lambda_{N_s}^S \) of \( P_{A(z)} \hat{R} P_{A(z)} \) and the eigenvalues \( \lambda_1^N \geq \ldots \geq \lambda_{p-N_s}^N \) of \( P_{\perp A(z)} \hat{R} P_{\perp A(z)} \), the ML height estimator is

\[
\hat{z}_{N_s} = \arg \min_{z \in \mathbb{C}^{N_s}} \alpha(z_{N_s}) \tag{8}
\]

with

\[
\alpha(z_{N_s}) = \left( \prod_{i=1}^{N_s} \lambda_i^S(z_{N_s}) \right)^{\frac{1}{p-N_s}} \sum_{i=1}^{p-N_s} \lambda_i^N(z_{N_s})^{p-N_s}. \tag{9}
\]

An algorithm based on alternating projections [8] is employed to solve this nonlinear, multimodal \( N_s \)-dimensional minimization problem.

**III. POLARIMETRIC MULTIBASELINE INSAR SPECTRAL ANALYSIS ALGORITHMS**

In this section, the spectral analysis techniques are extended to the fully polarimetric MB InSAR configuration. In this situation, the antennas not only receive the signals in diverse polarizations [10], [7], but emit the electromagnetic waves and receive the echo in polarimetric mode. The following adaptation to the fully polarimetric case not merely increases the number of observables, but especially finds the optimal polarization combination for height estimation. Furthermore, these algorithms allow examining the scatterers’ physical properties by analysis of their polarimetric behavior.

**A. Polarimetric MB InSAR Signal Model**

The signal model for multibaseline InSAR data described in section II has to be generalized to take polarization diversity into account. The polarimetric multibaseline interferometric SAR received signal for \( p \) sensors is modeled as:

\[
y(n) = \sum_{i=1}^{N_s} \sqrt{\tau_i} \exp(j\psi_i) b(z_{i1}, k_i) + \sum_{j=1}^{N_z} \sqrt{\tau_j} x_j(n) \odot b(z_{c_j}, k_j) + v(n). \tag{10}
\]

The observation vector \( y(n) \in \mathbb{C}^\hat{p} \), \( \hat{p} = 4p \), is a Gaussian random process with nonzero mean and covariance matrix \( \hat{R} \in \mathbb{C}^{\hat{p} \times \hat{p}} \), i.e. \( y(n) \sim N_{\mathbb{C}}(\mu, \hat{R}) \).

The main modification with respect to the single polarization model is the structure of the MB polarimetric interferometric (MBPI) steering vector \( b(z, k) \in \mathbb{C}^p \). It is a linear combination of several steering vectors \( a_{\gamma_i}(z) \in \mathbb{C}^p \), each of them associated to one particular polarization:

\[
b(z, k) = k_1 a_{\gamma_1}(z) + k_2 a_{\gamma_2}(z) + k_3 a_{\gamma_3}(z) + k_4 a_{\gamma_4}(z). \tag{11}
\]

The weighting coefficients \( k_i \in \mathbb{C} \) form a vector

\[
k = [k_1, k_2, k_3, k_4]^T \in \mathbb{C}^4 \tag{12}
\]

that may be interpreted as a scattering mechanism. This can be written in matrix notation as

\[
b(z, k) = B(z)k \tag{13}
\]

with the matrix of MBPI steering vectors \( B(z) \in \mathbb{C}^{\hat{p} \times 4} \)

\[
B(z) = [a_{\gamma_1}(z), a_{\gamma_2}(z), a_{\gamma_3}(z), a_{\gamma_4}(z)]. \tag{14}
\]

**B. Polarimetric MB InSAR MUSIC Algorithm**

If \( N_s \) is the assumed number of scatterers, the matrix of the noise eigenvectors is \( E_q \in \mathbb{C}^{\hat{p} \times q} \) with \( q = \hat{p} - N_s \). The spectrum of MUSIC for the fully polarimetric SAR configuration is

\[
\hat{P}_{MU}^{SP}(\gamma) = \frac{1}{\lambda_{\min}(B^H(z)E_qE_q^H)} \tag{15}
\]

with \( \lambda_{\min} \) the smallest eigenvalue of the \( 4 \times 4 \) Hermitian linear system

\[
B^H E_q E_q^H B_{\min} = \lambda_{\min} k_{\min}. \tag{16}
\]
The eigenvector $k_{\min}$ describes the physical features of the scatterer and permits a polarimetric analysis [11]. The linear system (16) must be of full rank, otherwise $\lambda_{\min} = 0$. This leads to an infinite spectrum (15) and the height cannot be determined. A necessary criterion for the linear system having full rank is $\hat{p} \geq N_s + 4$.

C. Polarimetric Multibaseline InSAR MLE

Let $C(z) = [B(z_1), \ldots, B(z_{N_s})]$ be the matrix of steering matrices (14) and the block diagonal polarization matrix $K \in C^{4N_s \times N_s}$

$$K = \text{diag} [k_1, \ldots, k_{N_s}]. \quad (17)$$

Using the abbreviation $D = C(z)K$, and keeping in mind that the matrix $D$ depends both on height $z$ and polarization $K$, the projection matrix $P_D \in C^{\hat{p} \times \hat{p}}$ onto the signal subspace is

$$P_D = D (D^H D)^{-1} D^H \quad (18)$$

and the orthogonal projection $P_D^\perp \in C^{\hat{p} \times \hat{p}}$ onto the noise subspace is

$$P_D^\perp = I - P_D. \quad (19)$$

Utilizing the decompositions of the matrices $P_D \hat{R} P_D$ and $P_D^\perp \hat{R} P_D$, the ML height estimator is

$$(\hat{z}_{N_s}, K) = \arg \min_{(z_{N_s}, K)} \beta(z_{N_s}, K) \quad (20)$$

with

$$\beta(z_{N_s}, K) = \delta(z_{N_s}, K) \varepsilon(z_{N_s}, K) \quad (21)$$

where

$$\delta(z_{N_s}, K) = \prod_{i=1}^{N_s} I_i^S(z_{N_s}, K) \quad (22)$$

and

$$\varepsilon(z_{N_s}, K) = \left( \frac{1}{\hat{p} - N_s} \sum_{i=1}^{\hat{p} - N_s} I_i^K(z_{N_s}, K) \right) \hat{p} - N_s. \quad (23)$$

IV. EXPERIMENTAL RESULTS

To demonstrate the performance of the algorithms introduced above, some results using different configurations are presented next. The height and physical characteristics of the dominant scatterers are determined by the single-baseline fully polarimetric MUSIC algorithm with a baseline of 10 m. Subsequently, the building layover problem is resolved using the dual-baseline fully polarimetric MLE.

A. Single-Baseline Polarimetric Height Estimation and Physical Feature Extraction

The fully polarimetric MUSIC method not only allows height estimation, but also a polarimetric analysis by means of the scattering vector $k_{\min}$, of the linear system (16). Three canonical scattering mechanisms are identified and assigned to the double bounce (DB), surface reflection (SR), and volume diffusion (VD) class [12]. While all these backscattering phenomena are present in the classification of the original data (Figure 1, top), the MUSIC method is able to eliminate the volume diffusion class (Figure 1, bottom), suggesting that the algorithm reduces the contribution induced by complex scenes including vegetation and man-made objects.

The height of buildings is estimated by the small baseline ($\approx 10$ m) fully polarimetric MUSIC method. The 3D image (Figure 2) shows a large scene of the Dresden dataset. It is color coded by the amplitude $|HH|$. Samples belonging to weakly correlated or shadowed areas are masked out by a criterion based on a threshold of the amplitudes and coherence. It has to be borne in mind that layover effects are still visible. Overall, the building height is well estimated, especially for edifices aligned in azimuth direction. Buildings with another orientation are partially masked out due to low backscattered power.

Figure 3 illustrates in a 3D view the physical characteristics of the dominant scatterer by means of the angle $\alpha_1$: blue represents surface reflection ($\alpha_1 \approx 0$), red double bounce ($\alpha_1 \approx \pi/2$). On the top of buildings, double bounce reflection prevails. This can be explained by particular roof structures or multipath effects generated by surrounding buildings. This argument is confirmed by average $\alpha_1$ values for the isolated building in the rear part of the scene.

B. Dual-Baseline Polarimetric Height Estimation and Physical Feature Extraction for Layover Solution

The height of up to two sources within one azimuth-range resolution cell is extracted by the dual-baseline polarimetric MLE with normal baselines of approximately 10 and 40 m. Subsequently, the physical properties of the reflectors are examined by calculating the $\alpha_1$ value of the associated MUSIC scattering vector $k_{\min}$. Figure 4 depicts the estimation for a building layover sample line where near range is on the left side. At far range, the height of a single scatterer is estimated that corresponds to the topography at the ground-wall interaction points. The quite high $\alpha_1$ values indicate double-bounce reflection. In the middle, the height retrieval is perturbed by a mixture of complex scattering mechanisms. For the samples inside the layover that are nearest to the sensor (left side), the elevations of two components are determined: The first at about 0 m is related to the ground topography, the second to the building roof. The indicator $\alpha_1$ attains average values. This means that two sources inside one resolution cell of building layover are separated and their heights and polarimetric characteristics are estimated.

V. CONCLUSION

This paper has introduced two spectral analysis techniques in their most general form: They can be employed for fully polarimetric MB InSAR configurations in all possible combinations. They optimize the polarizations for scatterer height estimation and permit to determine their physical behavior. The height of buildings and their physical properties are extracted by means of single-baseline polarimetric datasets. Using dual-baseline observations, the proposed techniques solve the layover problem by separating two contributions within one resolution cell, one related to the ground, the other...
to the building roof. In the future, the information extracted from different antenna locations will be synchronized and merged.

REFERENCES