Abstract—Estimating the number of backscattering sources is an important issue in analyzing multibaseline interferometric SAR data. This paper extends model order selection algorithms to process polarimetric multibaseline InSAR observations. These methods are applied to urban environments using fully polarimetric dual-baseline InSAR data of Dresden city acquired by DLR’s E-SAR system. Experimental results for single polarization and polarimetric dual-baseline InSAR set-ups are presented and discussed.

I. INTRODUCTION

The analysis of urban areas via SAR data at moderate resolution is a very difficult task [1], [2], [3] due to three main reasons: Firstly, radar images are geometrically distorted by layover and shadowing phenomena. Secondly, scattering patterns are very complex with various contributions within one resolution cell. Lastly, the speckle effect complicates SAR data analysis.

Estimating the number of backscattering sources, the model order, is a crucial step in characterizing multibaseline (MB) interferometric SAR signals. Recently, this detection problem has been examined in detail [4], [5], [6] investigating in particular the influence of the SAR speckle effect. This paper generalizes model order selection techniques to take polarization diversity into account. Section II describes a signal model for polarimetric MB interferometric SAR observations. In section III, the model order selection (MOS) algorithms that are based on information theoretic criteria (ITC) are adapted to this configuration. These methods are applied to urban environments using fully polarimetric dual-baseline InSAR data of Dresden city acquired by DLR’s E-SAR system. Experimental results for single polarization and polarimetric dual-baseline InSAR set-ups are presented and discussed in section IV.

II. POLARIMETRIC MULTIBASELINE INSAR SIGNAL MODEL

The polarimetric multibaseline InSAR received signal with \( p \) sensors can be modeled as

\[
y(n) = \mathbf{t} + c(n) + \mathbf{v}(n) \\
= \sum_{i=1}^{N_{c_t}} \sqrt{r_i} \exp(j\psi_i) \mathbf{b}(z_{t_i}, \mathbf{k}_{t_i}) + \sum_{j=1}^{N_{c_s}} \sqrt{r_j} x_j(n) \odot \mathbf{b}(z_{c_j}, \mathbf{k}_{c_j}) + \mathbf{v}(n) \tag{1}
\]

with \( n = 1, \ldots, N \), the number of looks \( N \), and the Schur-Hadamard product \( \odot \) (elementwise multiplication). The polarimetric observation vector \( \mathbf{y}(n) \in \mathbb{C}^{p} \), \( \tilde{p} = pN_{pol} \), is assumed to be a Gaussian random process with nonzero mean and covariance matrix \( \mathbf{R} \in \mathbb{C}^{p \times \tilde{p}} \), i.e. \( \mathbf{y}(n) \sim \mathcal{N}_{\mathbb{C}}(\mathbf{\mu}, \mathbf{R}) \). The coefficient \( N_{pol} \) attains values ranging from \( N_{pol} = 1 \) for conventional single polarization received data to \( N_{pol} = 4 \) for quad-polarized data. The first term, \( \mathbf{t} \), is highly coherent and can be associated to a deterministic or almost deterministic target [7]. The second contribution, \( c(n) \), represents the response of distributed environments including the SAR speckle effect as multiplicative noise [8]. The detection problem consists in estimating the total number of backscattering sources \( N_{s} = N_{s_t} + N_{s_c} \) where \( N_{s_t} \) and \( N_{s_c} \) are the number of scatterers related to the coherent and incoherent component, respectively. The additive white Gaussian noise \( \mathbf{v}(n) \in \mathbb{C}^{p} \) has zero mean and power \( \sigma_{v}^{2} \), i.e. \( \mathbf{v}(n) \sim \mathcal{N}_{\mathbb{C}}(0, \sigma_{v}^{2} \mathbf{I}) \). The reflectivity \( \tau \) and the height \( z \) of the scatterers as well as the complex argument \( \psi \) are considered to be deterministic unknown quantities. The single polarization multibaseline InSAR steering vector \( \mathbf{a}(z) \in \mathbb{C}^{p} \) for a general acquisition geometry is represented as

\[
\mathbf{a}(z) = [1, \exp(j\kappa_{zz}z), \ldots, \exp(j\kappa_{zp}z)]^T \tag{2}
\]
with the vertical wavenumber \( \kappa_z = \frac{4\pi}{\lambda} \frac{B^z}{\sin\theta} \). The structure of the MB polarimetric interferometric (MBPI) steering vector \( \mathbf{b}(z, k) \in \mathbb{C}^{\tilde{p}} \) is as follows: It is a linear combination of several steering vectors \( \mathbf{a}_s(z) \in \mathbb{C}^{\tilde{p}} \), each of them associated to one particular polarization:

\[
\mathbf{b}(z, k) = k_1 \mathbf{a}_{s_1}(z) + k_2 \mathbf{a}_{s_2}(z) + \ldots + k_\tilde{p} \mathbf{a}_{s_{\tilde{p}}}(z). \tag{3}
\]

The vector \( \mathbf{a}_{s_i}(z) \), for example, has the shape \( \mathbf{a}_{s_i}(z) = [\mathbf{a}^T(z), \mathbf{0}^T]^T \in \mathbb{C}^{\tilde{p}} \). The weighting coefficients \( k_i \in \mathbb{C} \) form a vector

\[
\mathbf{k} = [k_1, k_2, \ldots, k_{\tilde{p}}]^T \in \mathbb{C}^{\tilde{p}} \tag{4}
\]

that may be interpreted as a scattering mechanism.

The multiplicative noise \( \mathbf{x}_s(n) \in \mathbb{C}^{\tilde{p}} \) is assumed to be a Gaussian random vector with zero mean and covariance matrix \( \mathbf{C}_s = E\{\mathbf{x}_s(n)\mathbf{x}_s^H(n)\} \) where \( H \) denotes transpose, complex conjugate.

III. POLARIMETRIC MULTIBASELINE INSAR MODEL ORDER SELECTION

In this section, the model order selection techniques for resolving the polarimetric multibaseline InSAR detection problem are outlined emphasizing the differences to the single polarization scenario.

A. Data Model and Problem Formulation

To estimate the number of sources, Wax et Kailath [9] have proposed a model where the received signal vector \( \mathbf{y}(\cdot) \in \mathbb{C}^{\tilde{p}} \) is a superposition of a finite number of signals embedded in additive noise

\[
\mathbf{y}(n) = \sum_{i=1}^{N_s} \mathbf{b}(\varphi_i) \mathbf{s}_i(n) + \mathbf{v}(n) \tag{5}
\]

with \( n = 1, \ldots, N \), and the number of samples \( N \). The terminology is as follows: \( \mathbf{s}_i(\cdot) \) is a scalar complex waveform called the \( i \)th signal, \( \mathbf{b}(\varphi) \in \mathbb{C}^{\tilde{p}} \) a vector dependent on the unknown parameter vector \( \varphi \), of the \( i \)th source, and \( \mathbf{v}(\cdot) \in \mathbb{C}^{\tilde{p}} \) the additive noise.

We assume that the data has the following statistical properties: The signals \( \mathbf{s}_i(\cdot) \in \mathbb{C} \), \( i = 1, \ldots, N_s \), with \( N_s < \tilde{p} \), are assumed to be stationary and ergodic Gaussian random processes with zero mean and positive definite covariance matrix. The noise \( \mathbf{v}(\cdot) \in \mathbb{C}^{\tilde{p}} \) is assumed to be a stationary and ergodic Gaussian random vector, independent of the signal, with zero mean and covariance matrix \( \sigma_v^2 \mathbf{I} = \mathbb{C}^{\tilde{p} \times \tilde{p}} \) where \( \sigma_v^2 \) is the unknown noise power.

The objective is to estimate the number of sources \( N_s \) using a finite number of observations \( \{\mathbf{y}(1), \ldots, \mathbf{y}(N)\} \) that are assumed to be independent and identically distributed. This can be done by analyzing the structure of the covariance matrix: For a finite set of observations \( \{\mathbf{y}(1), \ldots, \mathbf{y}(N)\} \) the matrix form of this model is

\[
\mathbf{y}(n) = \mathbf{Bs}(n) + \mathbf{v}(n) \tag{6}
\]

with the matrix \( \mathbf{B} = [\mathbf{b}(\varphi_1), \ldots, \mathbf{b}(\varphi_{N_s})] \in \mathbb{C}^{\tilde{p} \times N_s} \) and the vector \( \mathbf{s}(n) = [s_1(n), \ldots, s_{N_s}(n)]^T \in \mathbb{C}^{N_s} \).

Under the above conditions, the data covariance matrix \( \mathbf{R} = E\{\mathbf{y}(n)\mathbf{y}^H(n)\} \in \mathbb{C}^{\tilde{p} \times \tilde{p}} \) is

\[
\mathbf{R} = \mathbf{BSB}^H + \sigma_v^2 \mathbf{I} \tag{7}
\]

where \( \mathbf{S} = E\{\mathbf{s}(n)\mathbf{s}^H(n)\} \in \mathbb{C}^{N_s \times N_s} \) is the covariance matrix of the signals.

The assumption that the vectors \( \mathbf{b}(\varphi_i) \) are linearly independent is equivalent to the fact that the matrix \( \mathbf{B} \) has full column rank, \( \operatorname{rank}(\mathbf{B}) = N_s \). Furthermore, assuming that the signal covariance matrix is nonsingular, \( \operatorname{rank}(\mathbf{S}) = N_s \), it follows that \( \operatorname{rank}(\mathbf{BSB}^H) = N_s \) and the \( \tilde{p} - N_s \) smallest eigenvalues of this matrix are all equal to zero. Let \( \lambda_1 \geq \cdots \geq \lambda_{\tilde{p}} \) be the eigenvalues of the covariance matrix \( \mathbf{R} \) in descending order. Then the smallest \( \tilde{p} - N_s \) eigenvalues \( \lambda_i = \sigma_v^2 \) for \( i = N_s + 1, \ldots, \tilde{p} \), which means that the number of sources can be retrieved by the multiplicity of the smallest eigenvalues.

The problem is that the covariance matrix is unknown and has to be estimated from a finite dataset that implies unequal eigenvalues with probability one. Furthermore, in the application of detecting the signals from polarimetric multibaseline InSAR data, the above described model for order selection and the MB POL-InSAR data model from section II differ in several ways: The coherent component leads to a received data vector with nonzero mean and the incoherent part comprises the speckle effect as multiplicative noise.

B. Information Theoretic Criteria and Model Order Selection

For a set of \( N \) observations, \( \mathbf{Y} = \{\mathbf{y}(1), \ldots, \mathbf{y}(N)\} \), and a family of models, i.e. a parameterized family of probability densities \( f(\mathbf{Y} | \mathbf{\theta}(k)) \), the task is to select the best fitting model.

The general information theoretic criterion can be expressed as

\[
\text{ITC}(k) = - \log f(\mathbf{Y} | \hat{\mathbf{\theta}}(k)) + g(\eta(k)) \tag{8}
\]

with the maximum likelihood estimate \( \hat{\mathbf{\theta}} \) of the parameter vector \( \mathbf{\theta} \) and the assumed number of sources \( k \). The first term is the log-likelihood of the maximum likelihood estimator of the model parameters. The bias correction \( g(\eta(k)) \) accounts for the difference between the estimated density \( f(\mathbf{Y} | \hat{\mathbf{\theta}}(k)) \) and the modeled density \( f(\mathbf{Y} | \mathbf{\theta}(k)) \). Then the number of sources \( N_s \) can be estimated via:

\[
\hat{N}_s = \arg \min_k \text{ITC}(k). \tag{9}
\]

Different choices of the penalty function lead to various approaches such as the Akaike information criterion, the minimum description length and the efficient detection criterion

\[
\text{EDC}(k) = - \log f(\mathbf{Y} | \hat{\mathbf{\theta}}(k)) + \eta(k) \log N. \tag{10}
\]

Let us assume the number of sources is \( k \in \{0, 1, \ldots, \tilde{p} - 1\} \). Let \( \lambda_1 \geq \cdots \geq \lambda_{\tilde{p}} \) and \( \mathbf{v}_1, \ldots, \mathbf{v}_{\tilde{p}} \) be the eigenvalues and eigenvectors, respectively, of the covariance matrix \( \mathbf{R} \). The model is described by the parameter vector \( \mathbf{\theta}(k) = (\lambda_1, \ldots, \lambda_k, \sigma_v^2, \mathbf{v}_1^T, \ldots, \mathbf{v}_k^T)^T \).
The log-likelihood of the maximum likelihood estimator of the model parameters is given by [9]
\[
\log f(Y|\hat{\theta}(k)) = (\hat{p} - k)N \log \left( \prod_{i=2}^{\hat{p}} \frac{\delta_{i}}{\sum_{i=1}^{\hat{p}} \delta_{i}} \right)
\]
with the eigenvalues \( l_1 \geq \cdots \geq l_{\hat{p}} \) of the sample covariance matrix
\[
\hat{R} = \frac{1}{N} \sum_{i=1}^{N} y(n)y^H(n).
\]

To calculate the number of free real-valued parameters, we have to determine the dimension of the space spanned by \( \theta(k) \): Since the eigenvalues of the covariance matrix are real-valued, they contribute \( k \) degrees of freedom (DoF). The \( k \) complex eigenvectors form an orthonormal basis of the matrix space \( \mathbb{C}^{\hat{p} \times k} \) with \( \dim(\mathbb{C}^{\hat{p} \times k}) = 2\hat{p}k \). The normalization decreases the DoF by \( 2k \), the mutual orthogonalization by \( k(k-1) \). Thus, the orthonormal eigenvalues span a space of dimension \( 2\hat{p}k - 2k - k(k-1) = k(2\hat{p} - 1 - k) \). The degrees of freedom for the multibaseline interferometric configuration with polarization diversity is
\[
\eta(k) = k + 1 + k(2\hat{p} - 1 - k) = k(2\hat{p} - k) + 1. \quad (13)
\]

The final information theoretic criterion is obtained by inserting (11) and (13) into (10).

A preprocessing technique to stabilize the variations of the small eigenvalues is diagonal loading
\[
\hat{R} = \hat{R} + \delta \sigma_{y}^2 I
\]
where \( \delta \) is the loading factor and \( \sigma_{y}^2 \) the additive noise power.

IV. EXPERIMENTAL RESULTS

The model order selection methods are evaluated using fully polarimetric dual-baseline InSAR data over Dresden city. Figure 1 shows a Pauli color-coded POLSAR image of a large scene that includes man made objects such as buildings, tree and grassland vegetation and surfaces like sports fields and bare soils.

A. Single Polarization Dual-Baseline InSAR MOS

First, the number of scatterers is determined using the single-polarization dual-baseline interferometric information employing the MOS technique and EDC without diagonal loading (Figure 2). The maximum number of detectable sources is two. Bare soils and shadows that are generated by buildings are partially assigned to the class of zero reflectors. Vegetated areas include one or two backscattering sources, whereas the building layover is associated to the class containing two or more signal components.

Figure 3 depicts the estimation result with diagonal loading and loading factor \( \delta = 10 \). Diagonal loading reduces considerably the model order. The building shadow is now clearly identified as an area without a detected signal, just like some surfaces such as the smooth sports field. The building layover that comprises two or more scatterers is noticeably surrounded by resolution cells where only one reflector is present.

B. Polarimetric Dual-Baseline InSAR MOS

To compare the single polarization and fully polarimetric InSAR MOS, the number of detected signal components is first restricted to two utilizing the polarimetric dual-baseline interferometric data (Figure 4). Globally, the results of the single polarization and fully polarimetric estimation resemble quite strongly. Especially for man made objects the polarimetric MOS tends to slightly estimate more reflectors than the single polarization approach and less scatterers over some vegetated areas.

To investigate the building layover in more detail, a subarea including edifices that are oriented in the sensor flight direction was selected. For display purposes, the maximum number of estimated sources was set to five. At the layover edges (Figure 5), the model order is lowest with one or two signal components, reaching its maximum of five in the middle of layover. Two-dimensional filtering causes a mixture of scattering mechanisms comprising single-bounce reflection from the ground and the roof, double-bounce from ground-wall interaction and possible multipath phenomena. To limit the influence of two-dimensional \( 5 \times 5 \) boxcar filtering, an adaptive filtering with a filter size of ten pixels was applied. It is evident (Figure 6) that this preprocessing reduces the model order, in particular inside the layover.

In the future, joint detection-estimation approaches [10] will be used to determine simultaneously the number of scatterers and their properties.

REFERENCES

Fig. 1. Pauli colorcoded POLSAR image.

Fig. 2. Single polarization dual-baseline InSAR MOS without diagonal loading.

Fig. 3. Single polarization dual-baseline InSAR MOS with diagonal loading.

Fig. 4. Polarimetric dual-baseline InSAR MOS with diagonal loading.

Fig. 5. Polarimetric dual-baseline InSAR MOS with $5 \times 5$ boxcar filtering.

Fig. 6. Polarimetric dual-baseline InSAR MOS with adaptive filtering.