The Distribution of Interferometric Phase Differentials and a Self-Initialising PolInSAR Classifier

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Abstract

This paper describes an unsupervised classifier for polarimetric interferometric SAR (PolInSAR) data. Expectation maximisation is used to estimate class parameters that maximise the probability of a dataset for a given number of classes. Polarimetric information, in the form of coherency matrices, and interferometric information, in the form of complex coherences, is taken into account. Phase differentials between complex coherences in different polarisation bases are used to make the classifier sensitive to the vertical structure of the scene under observation, and a distribution of such phase differentials is developed. The classifier is self initialising in that it does not rely on decompositions or thresholds. Classification results based on real data are presented and discussed.

1 Introduction

Unsupervised classification is often essential in the automated analysis of SAR remote sensing data. Classification results make data easier to interpret by users, and can serve as a starting point for automated analysis techniques that apply to homogeneous regions of a particular type of land cover. The classification of PolInSAR data, in particular, is important, as this type of data has been shown to contain a wealth of information that is not obtainable by polarimetry or interferometry alone. Of special interest is the fact that PolInSAR data contains information concerning the vertical structure of the scene under observation. This type of information is potentially useful in the analysis of forest, agricultural and urban areas.

A number of PolInSar classifiers have been proposed in the past, including classifiers that are able to distinguish different types of forest with a high degree of accuracy [1] [2]. These classifiers have demonstrated that interferometric information, in the form of (optimised) coherences, is essential in the segmentation of data containing volumetric structures. Interferometric information also makes the discrimination of man made structures possible in cases where polarimetric information alone is ambiguous [3]. Most of the methods proposed to date have, however, some shortcomings.

The 6 × 6 covariance matrix is commonly employed in the iterative determination of class parameters. As these matrices contain the absolute phase of the complex coherence, the classification is sensitive to topography, which is undesirable in the identification of land cover types. In addition, the degree of coherence, which is implicit in these matrices, is a biased estimator of the true coherence [4]. This effect leads to lower contrast and, as a consequence, lower classification sensitivity over regions with low coherence. Finally, these classifiers usually rely on arbitrary thresholds for initialisation. Although these initialisations perform well in practice, it is conceivable that some datasets have a structure that is not well represented by the particular choice of thresholds. Classes that extend to both sides of a threshold can be misrepresented in the classification result.

Figure 1: The PolInSAR dataset used in the evaluation of the classifier. The scene features part of the DLR at Oberpfaffenhofen, Germany, and was acquired, in repeat-pass mode and with a baseline of approximately 10m, by the E-SAR sensor. The dataset is pre-processed by removing the flat earth phase and applying a range spectral filter.

The classifier presented in this paper aims to address some of these difficulties. It takes into account the distributions of polarimetric class content, the absolute degree of interferometric coherence in several polarisation bases, and phase differentials between complex interferometric coherences in several polarisation bases. By treating the degree of coherence separately, it is possible to make unbiased estimates of the true coherence in a class, and the information inherent in phase differentials is both independent of topography and contains information relating to the vertical structure of the scene under observation.

Section 2 introduces the distributions relevant to the classifier, and contains the derivation of the distribution of interferometric phase differentials. Section 3 describes how these distributions can be used for classification by com-
bining them in an expectation maximisation framework. Section 4 discusses the self-initialising nature of the classifier. In section 5, segmentation results for real PolInSAR data are presented and discussed, and section 6 contains conclusions.

![Figure 2: The distribution of interferometric phase differentials.](image)

## Distributions

PolInSAR data are obtained by coherently combining measurements from two polarimetric SAR acquisitions. Each polarimetric dataset is processed to yield a scattering matrix for each resolution cell in the image. The matrix contains the complex co- and cross-polar responses of targets within the resolution cell, and can, in the monostatic case and in the Pauli representation, be expressed as a vector $k_i$:

$$k_i = \frac{1}{\sqrt{2}} [S_{HH,i} + S_{VV,i}, S_{HH,i} - S_{VV,i}, 2S_{HV,i}]^T$$

The vectors $k_1$ and $k_2$ of each polarimetric dataset are concatenated to obtain the vector $k$, which is used to compute the interferometric coherency matrix $T$:

$$T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \langle kk^\dagger \rangle$$

where $\begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$

Averaging is accomplished by multilooking or spatial averaging with a speckle filter. The matrix $T_{12}$ can be used to estimate the complex interferometric coherences $(\gamma_1, \gamma_2, \gamma_3)$ in the Pauli polarisations:

$$\gamma_i = \frac{T_{12}(i,i)}{\sqrt{T_{11}(i,i)T_{22}(i,i)}}$$

In the following treatment, the amplitude and phase of the interferometric coherence are treated separately, and $\gamma_i$ is written $d_i \exp(i \phi_i)$.

The distributions governing polarimetric coherency matrices and complex interferometric coherences are known. Goodman [5] showed that coherency matrices in a Gaussian homogeneous area follow the complex Wishart distribution:

$$p(T_{xx}|T_0, L) = \frac{|T_0|^{-L}L^{Lq} |T_{xx}|^{-L-q} Z(L,q)}{Z(L,q)} e^{(-LT_{xx}^\dagger T_0^{-1}T_{xx})}$$

where $T_0 = E[T_{xx}], L$ is the number of looks, $q$ is the length of scattering vectors underlying $T_{xx}$ and $Z(L,q)$ is a normalisation factor. The mean of $T_{xx}$ over a homogeneous area is an unbiased estimator of $T_0$.

The distribution of the complex interferometric coherences $\gamma_i$ was developed by Touzi and Lopes et al. The results include distributions over the degree of coherence $d_i$ as well as the interferometric phase $\phi_i$ [4]:

$$p(d_i|D_i, L) = 2(L - 1)(1 - D_i^2)^{\frac{L-1}{2}} \times 2F_1(L, L; 1; D_i^2 d_i^2) \quad L > 2$$

$$p(\phi_i|\alpha_i, D_i, L) = \frac{(1 - D_i^2)^{\frac{L-1}{2}}}{2\pi} \left[ \frac{3}{2} F_2(1, L; L; \frac{1}{2}; L; z_i^2) + \frac{\Gamma(\frac{L+1}{2})\Gamma(\frac{L+3}{2})}{\Gamma(\frac{L+2}{2})} \frac{3}{2} F_2(\frac{3}{2}, L; L+\frac{1}{2}; L; \frac{1}{2}; z_i^2) \right]$$

$D_i$ is the true coherence of the homogeneous area in polarisation $i$, $\alpha_i$ is the expected phase of the coherence, and $3F_2$ denotes the generalised hypergeometric function. The mean phase is an unbiased estimator of $\phi_i$. In the case of the degree of coherence $d_i$, however, Touzi et al. have shown that the mean $\langle d_i \rangle$ overestimates the true coherence $D_i$:

$$\langle d_i \rangle = \frac{\Gamma(L+1)\Gamma(1-D_i^2)}{\Gamma(L+2)} 3F_2 \left( \frac{3}{2}, L; L + 1, 1; D_i^2 \right)$$

When the degree of coherence is estimated over a homogeneous area, equation (5) must be inverted to remove the bias in $\langle d_i \rangle$.

The absolute phase of the interferometric coherence $\phi_i$ is not useful in the context of classification, as it depends primarily on the topography of the scene under observation. This is not the case for interferometric phase differentials $\Delta \phi_{ij}$ = $\phi_i - \phi_j$. These differentials are known to contain information concerning the vertical structure of the imaged scene. Before this type of information can be introduced into a classifier, it is necessary to derive the distribution of phase differentials $\Delta \phi_{ij}$.

Given a homogeneous area with interferometric phases $\phi_i$ and $\phi_j$ and degrees of coherence $D_i$ and $D_j$ for polarisations $i$ and $j$ respectively, the probability of observing interferometric phases $\phi_i$ and $\phi_j$ is given by

$$p(\phi_i, \phi_j|\alpha_i, \alpha_j, D_i, D_j, L) = p(\phi_i|\alpha_i, D_i, L)p(\phi_j - \Delta \phi_{ij}|\alpha_j, D_j, L)$$

This distribution can be marginalised to obtain the desired distribution of $\Delta \phi_{ij}$ by integrating over $\phi_i$. Writing $\Delta \alpha_{ij} = \alpha_i - \alpha_j$, the distribution can be calculated as follows.

$$p(\Delta \phi_{ij}|\alpha_i, \Delta \alpha_{ij}, D_i, D_j, L) = \int_0^{2\pi} p(\phi_i|\alpha_i, D_i, L) \times p(\phi_i - \Delta \phi_{ij}|\alpha_i, \Delta \alpha_{ij}, D_j, L) \, d\phi_i$$

An analytic solution to the convolution, albeit not in closed form, was given in [6]. In practice it is considerably more efficient to compute the distribution by numerically convolving two distributions, computed using equation (4), in the frequency domain. Also, it can be shown that the mean interferometric phase differential $\langle \Delta \phi_{ij} \rangle$ is an unbiased estimator of $\Delta \alpha_{ij}$. Figure 2 shows the distribution of $\Delta \phi_{ij}$ for degrees of coherence $D_i = 0.4$ and $D_j = 0.75$, and a mean interferometric phase differential $\Delta \alpha_{ij} = 0.6\pi$ for several values of $L$. 

3 Expectation Maximisation

Expectation maximisation (EM) is a general method for obtaining a set of model parameters that maximise the likelihood of a given dataset in the presence of hidden variables. In the case of classification, a hidden variable assigns pixels to classes. Once the correspondence between pixels and classes is known, it becomes comparatively easy to estimate the class parameters. Conversely, once the class parameters are known, it becomes comparatively easy to assign pixels to classes. Difficulties arise because neither information is available. EM is an iterative process in which both types of information are acquired simultaneously.

It can be shown that, in a very general sense, expectation maximisation can be broken down into two steps. The first step, the expectation or E step, consists of computing the probability density over hidden variables based on an initial estimate of the model parameters. The second step, the maximisation or M step, uses this distribution to obtain a refined estimate of the model parameters. For the purposes of classification, each class \( k \) is described by a parameter vector \( \theta^{(k)} \), containing an average polarimetric coherency matrix \( T_{0}^{(k)} \), the estimated degrees of coherence \( D_{ij}^{(k)} \), 1 \( \leq i \leq 3 \), in the three Pauli polarisations, and the three average interferometric phase differentials \( \Delta \alpha^{(k)}_{12} \), \( \Delta \alpha^{(k)}_{23} \) and \( \Delta \alpha^{(k)}_{31} \).

The probability of observing a pixel \( p \) with polarimetric coherency matrix \( T_{xx}^{(k)} \), degrees of coherence \( \alpha_{ij}^{(p)} \), 1 \( \leq i \leq 3 \), and interferometric phase differentials \( \Delta \phi_{ij} \) in class \( k \) is defined as

\[
p(p|\theta^{(k)}) = p(T_{xx}^{(p)}|T_{0}^{(k)}, L) \left( \prod_{i} p(D_{ij}^{(p)}|D_{ij}^{(k)}, L) \right) \times \left( \prod_{ij} p(\Delta \phi_{ij}^{(p)}|\Delta \alpha^{(k)}_{ij}, D_{ij}^{(k)}, D_{ij}^{(k)}, L) \right)
\]

(7)

\( T_{xx}^{(k)} \) can be taken as one of the matrices \( T_{11} \) or \( T_{22} \), or their mean. In the latter case, the number of looks \( L \) in equation (2) must be adjusted accordingly. The assumptions of independence implicit in this use of the elementary distributions do, strictly speaking, violate the hypothesis of Gaussian surfaces. The inaccuracies thus introduced are, however, considered to be outweighed by the advantages of this formulation (namely the ability to explicitly use phase differentials, and to invert the bias in the estimated coherence).

The E step consists of computing \( \zeta^{p}_{k} \), the normalised probability that pixel \( p \) belongs to class \( k \), for all \( p \) and \( k \). Assuming that \( p(\theta^{(k)}) \) is flat, \( \zeta^{p}_{k} \) is given by

\[
\zeta^{p}_{k} = \frac{p(p|\theta^{(k)})}{\sum_{k} p(p|\theta^{(k)})}
\]

(8)
The M step maximises the likelihood of the dataset by refining the parameter vector \( \theta^{(k)} \), taking into account the likelihoods of class membership \( \zeta^{p}_{k} \). The following update rules can be shown to achieve this maximisation.

\[
T_{0}^{(k)} = \frac{1}{N_{k}} \sum_{p} \zeta^{p}_{k} T_{xx}^{(p)}
\]

(9)

\[
D_{ij}^{(k)} = \text{INV} \left[ \frac{1}{N_{k}} \sum_{p} \zeta^{p}_{k} D_{ij}^{(p)} \right]
\]

(10)

\[
\Delta \alpha^{(k)}_{ij} = \frac{1}{N_{k}} \sum_{p} \zeta^{p}_{k} \Delta \phi_{ij}^{(p)}
\]

(11)

where \( N_{k} = \sum_{p} \zeta^{p}_{k} \) and the operator \( \text{INV}[\ldots] \) indicates that the true degree of coherence is obtained from the mean by numerically inverting equation (5).

4 Self-Initialisation

Typically, unsupervised classification algorithms use decompositions and, to some extent arbitrary, thresholds to initialise class parameters before an iterative refinement process can begin. Although this tends to work well in practice, there is no guarantee that the structure of a dataset suits the particular thresholds chosen. Problems generally arise when a class straddles one or more thresholds in feature space.

The approach chosen for this classifier is to gradually introduce new classes into the classification result. Given a number of desired classes \( N_{C} \), and the number of EM iterations between the introduction of new classes \( N_{T} \), the algorithm can be summarised as follows.

\[
\begin{align*}
\theta^{(1)} := & \text{Arbitrary Initialisation} \\
\text{FOR } k &= 1 \text{ to } N_{C} \\
\text{FOR } i &= 1 \text{ to } N_{T} \\
\zeta^{p}_{k} &= \text{E step} \\
\theta^{(1...k)} := & \text{M step} \\
\text{END} \\
\text{Initialise } \theta^{(k+1)} \\
\text{END}
\end{align*}
\]

The problem of initialisation is reduced from simultaneously determining suitable initialisation parameters for all classes \( 1 \ldots k \) to determining suitable parameters for a single class \( k \) based on a classification with classes \( 1 \ldots k-1 \). To determine the parameters of a new class \( k \), a preliminary classification result based on the previously established \( k-1 \) classes is computed. In this result, pixel \( p \) is assigned the class \( x \) that maximises \( \zeta^{p}_{x} \).

To assess how well a particular class \( y \) represents the pixels it has been assigned, its average log likelihood \( L_{y} \) is computed. If \( C_{y} \) is the set of pixels that have been assigned to class \( y \) in the preliminary result, and \( |C_{y}| \) is the number of such pixels, the average log likelihood of class \( y \) is given by

\[
L_{y} = \frac{1}{|C_{y}|} \sum_{p \in C_{y}} \log p(p|\theta^{(y)})
\]

(12)

Classes with low \( L_{y} \) are considered to comprise more than one homogeneous region. The class with the lowest log likelihood, class \( c \), is used to initialise the new class \( k \). The pixel \( p \in C_{c} \) with the highest \( \zeta^{p}_{c} \), is used as a seed point for the new class \( k \). This strategy is preferred over the alternative of using seed pixels with minimal \( \zeta^{p}_{c} \), as such pixels...
are frequently point targets or part of highly reflective surfaces with low SNR.

\[
T_0^{(k)} = T_{xx}^{(k)} \quad D_i^{(k)} = \text{INV} \left[ d_i^{(p)} \right] \\
\Delta \phi_{ij}^{(k)} = \Delta \phi_{ij}^{(p)}
\]

(13)

Figure 3: A classification result using only polarimetric information \( (N_C = 10) \)

Figure 4: A classification result using polarimetric and interferometric information \( (N_C = 10) \)

5 Results and Discussion

Figure 3 shows the classification obtained when the proposed self-initialising classifier is applied to the polarimetric coherency matrix \( T_{xx} \) only. In this case, the \( E \) step does not take into account the distributions over interferometric coherences. Although there is no guiding initialisation, the classifier is clearly able to distinguish the different basic types of land cover. Buildings, forest and agricultural areas are separated.

Figure 4 shows classification result in which polarimetry, degrees of coherence and interferometric phase differentials are taken into account. The buildings in the centre and the lower left of the image are identified correctly, where polarimetric information alone appears to have been insufficient and incorrect classifications as forest have occurred.

Figure 5: A classification based on interferometric phase differentials only \( (N_C = 2) \)

Figure 5 illustrates the information content inherent in the interferometric phase differentials. The classification result with two classes was obtained using only the distribution over interferometric phase differentials in the \( E \) step. The classes clearly differentiate regions with a vertical structure, such as forest and buildings, from flat surfaces.

6 Conclusion

An unsupervised classifier for PolInSAR data was described. The classifier uses expectation maximisation to derive a classification result that maximises the probability of the input dataset for a given number of classes. The expectation maximisation procedure is based on the previously established distributions of the polarimetric coherency matrix and the degree of interferometric coherence. The distribution of interferometric phase differentials is derived, and is applied to in the context of classification to achieve sensitivity regarding the vertical structure of the scene under observation.

References


