SAR Processing with Motion Compensation using the Extended Wavenumber Algorithm

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Abstract

Modern Synthetic Aperture Radar (SAR) systems are continuously developing into the direction of higher spatial resolution and new modes of operation. This requires the use of high bandwidths, combined with wide azimuthal integration intervals. For focusing such data, a high quality SAR processing method is necessary, which is able to deal with more general sensor parameters. Wavenumber domain (Omega-K) processing is commonly accepted to be an ideal solution of the SAR focusing problem [1]. However, it is mostly applicable on spaceborne SAR data where a straight sensor trajectory is given. In case of airborne SAR data, wavenumber domain processing has certain limitations in performing high-precision motion compensation.
In this paper, a detailed description of the motion errors in the wavenumber domain, as well as a motion compensation technique in this domain is formulated. The correction of the motion errors in the two dimensional spectral domain can result in very accurate second order motion compensation. This procedure can also be combined with a 2D sub-aperture technique, which results in a fully azimuth-frequency adaptive block processing scheme. The reason why the wavenumber MoCo can be very critical especially in low frequency-widebeam and high squinted SAR data, is that in these cases wavelength dependent corrections become mandatory.

1 Introduction

1.1 Importance of the motion compensation procedure

A crucial problem in most airborne SAR sensors is the compensation of motion errors, induced by atmospheric turbulence (i.e. the compensation of changes of the platform velocity vector in orientation and/or in magnitude). If not corrected, the image quality will considerably degrade [2].
Airborne sensors, in contrary to spaceborne sensors, always show deviations from the ideal flight track. SAR imaging from such unstable platforms requires an accurate measurement of the antenna position during the flight and a modified processing scheme, which takes into account the non-linear movement of the sensor.

1.2 The ECS and Wavenumber Domain approach for motion compensation

Chirp Scaling (CS) algorithm [3] was developed mainly to avoid interpolations, which are necessary when we had to deal with strong range-cell migration data (e.g. when wide azimuth beam data have to be processed). An improved version of the CS was the ECS algorithm [2], which was developed originally for processing airborne data with strong motion errors and variable Doppler centroid in range or/and in azimuth direction.
On the other hand, in the Omega-K algorithm [4], [5], due to its processing architecture a high precision motion correction cannot be introduced in its processing scheme. Compared with the ECS two-step motion compensation (MoCo), only one step MoCo can be applied [6]. In its very basic implementation it starts normally with range compression, allowing a first order MoCo. However, the range dependent part of the MoCo, which can only be applied after the correction of RCM, is not possible in Omega-K [6].

1.3 Extended Wavenumber Algorithm (Extended Omega-K)

Several processing schemes, which try to compensate this problem, using different approaches, can be found in the literature [7], [8]. The technique described in [6] for example is based on the Taylor approximation of the phase in the two dimensional spectral domain performs a scaling of the data in range by a scaled inverse range FT. In contrary the EOK algorithm, first proposed in [9], extends the basic Omega-K processing procedure and proposes a modified Stolt mapping in order to apply a robust and high accurate

MoCo algorithm even in case of high squint angles and long synthetic apertures.
The MoCo procedure in the EOK algorithm [9] is performed in two different steps. A first order MoCo takes place right just after or before the range compression and a second order MoCo after the Modified Stolt mapping, following in principle the same MoCo correction procedure as in ECS.
A problem of the Extended Omega-K approach appears in case of high squint and wideband SAR data processing, because of the range and wavenumber dependence of the motion errors. A solution of this problem is presented in this paper using a subaperture technique.

2 Extended Wavenumber Domain Algorithm

2.1 Extended Omega-K processing steps

The main processing steps of EOK, as analytically described in [2], are:

i) First order Motion Compensation
ii) 2D FFT
iii) Range compression
iv) Modified Stolt Mapping
v) 2D IFFT
vi) Second Order Motion Compensation
vii) Azimuth FFT
viii) Azimuth Compression
ix) Azimuth IFFT

Range compression of the data is performed if a chirped pulse has been transmitted. After the modified Stolt mapping and the 2D IFFT are performed the data are RCM corrected but still azimuth unfocused.
Important to note is that the original formulation of the Omega-K algorithm does not allow for the second order motion compensation with enough accuracy. This is due to the fact that after the Stolt mapping all the SAR data in the signal domain will have the azimuth chirps with the same length and Doppler rate (i.e. no range dependence). This leads to a mismatch between the SAR data and the motion compensation phase correction to be applied. The EOK algorithm with a modified Stolt interpolation and with a subaperture approach for motion compensation represents one accurate solution for this limitation of the standard Omega-K algorithm.

2.1 Modified Stolt Mapping

The modified Stolt mapping, introduced to the EOK algorithm [2], provides the possibility for improved second order motion compensation. In the conventional Omega-K algorithm, the Stolt mapping procedure [11] corresponds to an interpolation, which transforms the initial wavenumber domain (k_x, ω) to (k_x, k_r) where:

$$k_r = \sqrt{\left(\frac{\omega+\omega_o}{c/2}\right)^2 - k_x^{\,2}} - \frac{\omega_o}{c/2} \qquad (1)$$

ω_o is the center frequency, c is the speed of light and k_x is the Doppler wavenumber ($k_x = 2*\pi*f_D/v$, f_D is the Doppler and v is the platform velocity).
After the Stolt Mapping further motion compensation cannot be performed due to the fact that the azimuth chirps don't have any range dependence more (i.e. constant length and Doppler rate).
For this reason, a modified Stolt mapping [2] is performed:

$$k_r = \sqrt{\left(\frac{\omega+\omega_o}{c/2}\right)^2 - k_x^{\,2}} - \sqrt{\left(\frac{\omega_o}{c/2}\right)^2 - k_x^{\,2}} \qquad (2)$$

The data are not azimuth focused and a second motion compensation step can be integrated. From the above equation it becomes evident that the Modified Stolt mapping introduces also a wavelength dependent effect to the azimuth signal (and also to the motion errors). An interpretation of this effect as well as a possible solution is proposed in the following sections.

2.2 Wavelength dependent nature of motion errors

In this section a formulation of the problem is performed, based on the signal analysis of a target's response before and after the modified Stolt mapping.
The response of a point target in the signal domain can be expressed as [11]:

$$\delta(x - x_o, r - r_o) \rightarrow \qquad (3)$$
$$g_o\left(t - \frac{2R(x-x_o;r_o)}{c}\right) exp\left(-j\omega_o \frac{2R(x-x_o;r_o)}{c}\right)$$

where x_o is the azimuth position of the target, r_o is the range of closest approach, $R(x;r_o) = \sqrt{r_o^2 + x^2}$ and $g_o(.)$ is the complex envelope of the transmitted signal. Without loss of generality we choose $x_o = 0$ and the 2D response in the ω-k_x wavenumber domain, using the method of stationary phase [11], is:

$$PP(k_x,\omega) \cong \exp\left[-jr_o \cdot \sqrt{\left(\frac{\omega+\omega_o}{c/2}\right)^2 - k_x^{\,2}}\right] \qquad (4)$$

while the geometry of the signal spectrum is characterized by the following relation [1]:

$$\frac{x}{\sqrt{r_o^{\,2}+x^2}} = -\frac{k_x}{\frac{\omega+\omega_o}{c/2}} \Rightarrow k_x = -\sin\theta \cdot \frac{\omega+\omega_o}{c/2} \qquad (5)$$

$$\sin\theta = \frac{x}{\sqrt{r_o^{\,2}+x^2}} \qquad (6)$$

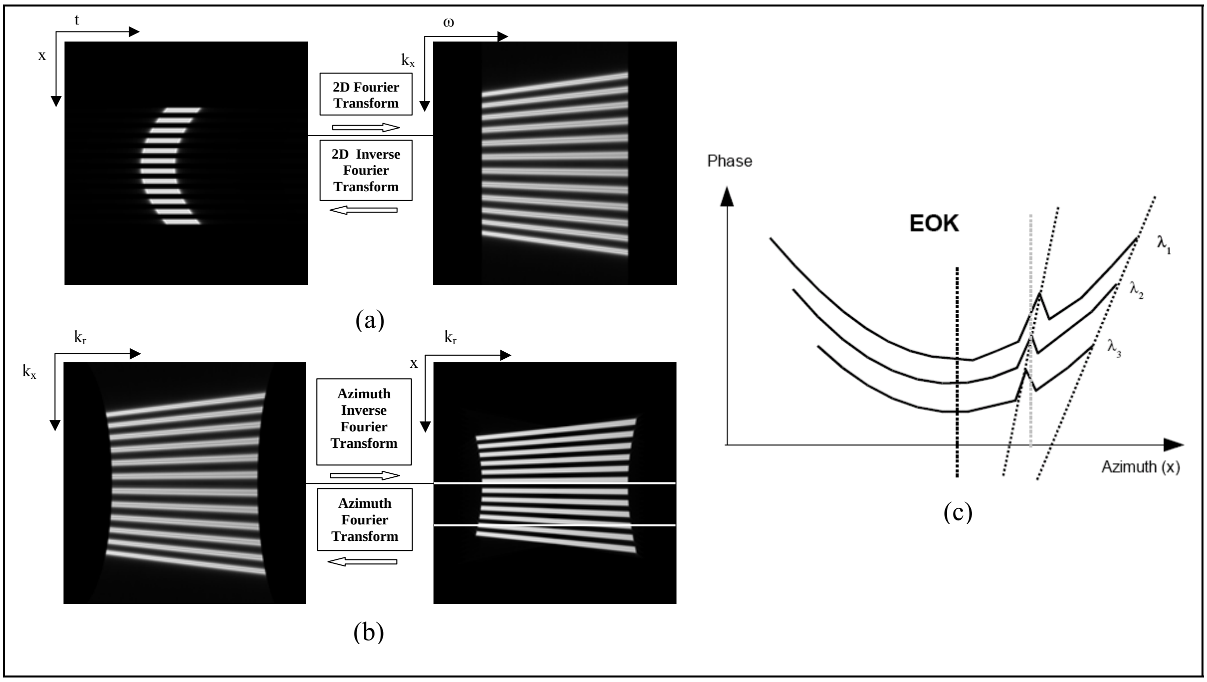

Fig. 1 (a) Correspondence between the different aspect angles (left) and the straight lines in the signal spectrum (right), (b) 2D spectrum (left) and azimuth - 'range frequency' (right) of the target's response after modified Stolt mapping (simulation parameters: 120 MHz center frequency, 30 MHz bandwidth, PRF = 100 Hz and –25° to 25° aspect angle) and (c) wavelength dependent position shift of the motion errors in the phase response.

From the above equation becomes evident that there is an one-one correspondence between the angle θ and the parametric straight line, in the wavenumber domain, given by **Eq. 5**. This correspondence is also demonstrated in **Fig 1a**.

It must be noted, that because the deviation from the nominal track changes as the platform moves, the deviation is only a function of the target's range and aspect angle but also of azimuth. As a consequence, different aspect angles (equivalently azimuth positions for a specific target) of the aperture correspond to different deviations. These lines can be characterized as *iso-MoCo* lines.

The modified Stolt mapping is performed by the change of variable given by **Eq. 2**. The response in the modified k_r-k_x wavenumber domain is:

$$PP_{\mathrm{mod}-SM}(k_x,\omega) \cong \exp(-jr_o \cdot k_r)\exp\left[-jr_o \cdot \sqrt{\left(\frac{\omega_o}{c/2}\right)^2 - k_x^{\,2}}\right] \tag{7}$$

and the straight line (see **Eq. 5**) that corresponds to a certain angle becomes (after the change of variable) a parametric curve defined by the following equation:

$$k_r = k_x \cdot \cot\theta - \sqrt{\left(\frac{\omega_o}{c/2}\right)^2 - k_x^{\,2}} \tag{8}$$

The different curves in the 2D spectrum domain are shown in **Fig 1b**. Furthermore, the image of the response in the k_r-azimuth domain is illustrated. To return back to the signal domain, only an inverse FT in the k_r direction is needed. After the inverse FT, targets are separated in range. But a problem appears in every target's response because of the way the inverse FT in k_r is calculated. This effect can be verified by observing that every horizontal line away from the azimuth center of the target truncates curves corresponding to different aspect angles. As a result of this, in the signal domain, an integrated error from an interval of the target's illumination time is accumulated to the target's final signal domain response (see **Fig 1c**).

This effect becomes even more important in high squint processing, because an integrated motion error from the entire target's illumination time is accumulated. It must be noted that this wavelength dependent position of the motion errors exists also to the initial raw data, but the second term of the phase response after the Stolt mapping (see **Eq. 7**) is not k_r dependent, in contrary to the phase response before the Stolt mapping. Consequently, the expansion is correct for the center frequency ($k_{ro} = 2\ \omega_o/c$) but not for other k_r.

2.3 Solution of the problem using sub-aperture technique

Block processing technique can be performed to the data in order to compensate the above-described effect. Based on this technique, for every azimuth-range position, the motion errors from different angles are unwrapped and then compensated.

After performing a 2D FFT to every block of the image, the parameter θ of the curve that passes from every *(k_x, k_r)* point is calculated from **Eq. 8**:

$$\cot\theta = \frac{k_r + \sqrt{\left(\frac{\omega_o}{c/2}\right)^2 - k_x^2}}{k_x} \quad (9)$$

Afterwards, the k_x', which is the k_x where the curve (see **Eq. 8**) crosses the horizontal axis ($k_r = 0$) is calculated:

$$k_x' = \frac{\omega_o}{c/2} \frac{k_x}{\sqrt{k_x^2 + \left(k_r + \sqrt{\left(\frac{\omega_o}{c/2}\right)^2 - k_x^2}\right)^2}} \quad (10)$$

In case of high squint, the above equation becomes:

$$k_x' = k_{xd} + sin(\theta_{squint}) \cdot cos(\theta_{squint}) \cdot k_r \quad (11)$$

Where k_{xd} is the k_x wavenumber corresponding to the squint angle and θ_{squint} is the squint angle.
The k_x and k_x' both correspond to two different azimuth positions (according to stationary phase theorem). The deviation (function of the azimuth position) for every point of the wavenumber domain (k_x, k_r) corresponds to a specific azimuth position. This specific azimuth position corresponds to the sum of the reference azimuth position x' of the specific block and the time difference between the previously described azimuth positions. Furthermore, it can be expressed as the deviation, which corresponds to the reference azimuth position shifted by the following time interval:

$$time\_difference = r' \cdot \left(\frac{k_x}{\sqrt{k_{ro}^2 - k_x^2}} - \frac{k_x'}{\sqrt{k_{ro}^2 - k_x'^2}} \right) \quad (12)$$

$$k_{ro} = \frac{\omega_o}{c/2} \quad (13)$$

where r' is the reference range position of the specific block.
Accurate motion compensation is achieved by performing the above procedure for every block and every point (k_x, k_r) of the 2D spectrum.

3 Conclusions and Future Work

A detailed description of the motion errors position in the wavenumber domain as well as a motion compensation technique in this domain was analytically formulated in this paper. It was demonstrated, that the correction of the motion errors in the two-dimensional spectral domain could result in very accurate 2nd order motion compensation. This procedure was combined with a 2D sub-aperture technique resulting in a fully azimuth-frequency adaptive block processing scheme. Although the proposed MoCo algorithm successfully compensates in several SAR system operational modes, the use of sub-aperture techniques increases the computational burden and requires extra precaution to avoid unwanted phase jumps at the boundaries of the sub-apertures. A new version of the algorithm, without the use of sub-aperture, is currently under development. Instead of correcting fully the problem in the wavenumber domain, a correction of the expansion for a reference range is performed, using a suitable phase function.

4 Literature

[1] C. Cafforio, C. Prati, and F. Rocca: "SAR Data Focusing Using Seismic Migration Techniques", IEEE Trans. on Aerosp. and Elec. Sys., Vol. 27, No. 2, pp. 194-207, 1991.

[2] A. Moreira, J. Mittermayer and R. Scheiber: "Extended Chirp Scaling Algorithm for Air- and Spaceborne SAR Data Processing in Stripmap and ScanSAR Imaging Modes", IEEE Trans. on Geosc. and Rem. Sens., Vol. 34, pp. 1123-1136, Sep. 1996.

[3] R.K. Raney, H. Runge, R. Bamler, I. Cumming and F. Wong: "Precision SAR Processing without interpolation for range cell migration correction", IEEE Trans. on Geosc. and Rem. Sens., Vol. 32, pp. 786-799, July 1994.

[4] G. Franceschetti and G. Schirinzi "A SAR processor based on two dimensional FFT codes", IEEE Trans. On Aerosp. Elec. Sys., Vol. 26, pp. 356-366, Mar. 1990.

[5] R. Bamler: "A comparison of range-Doppler and wavenumber domain SAR focusing algorithms", IEEE Trans. on Geosc. and Rem. Sens., Vol. 30, No. 4, pp. 706-713, July 1992.

[6] C. Cafforio, C. Prati, and F. Rocca: "Full Resolution Focusing of SEASAT SAR Images in the Frequency-Wavenumber Domain", Int. Journal of Rem. Sens., Vol. 12, pp. 491-510, 1991.

[7] G. Fornaro: "Trajectory Deviations in Airborne SAR: Analysis and Compensation", IEEE Trans. on Aerosp. and Elec. Sys., Vol. 35, No. 3, pp. 997-1009, July 1999.

[8] R. Lanari: "A new method for the comparison of the SAR range cell migration based on the chirp Z-transform", IEEE Trans. on Geosc. and Rem. Sens., Vol. 33, pp. 1296-1299, Sept. 1995.

[9] A. Reigber, A. Potsis, E. Alivizatos, N. Uzunoglu and A. Moreira, "Wavenumber Domain SAR Focusing with Integrated Motion Compensation", IGARSS 2003.