Wavenumber Domain SAR Focusing with Integrated Motion Compensation

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***Abstract*— In this paper a new SAR data processing algorithm denoted with *Extended Omega-K* (EOK) is analytically presented and formulated. EOK algorithm combines the advantages of the high accurate focusing of the wavenumber domain algorithms with high precision motion compensation. The new EOK algorithm integrates a two-step range adaptive motion compensation correction in the general formulation of the wavenumber domain algorithm, leading to a new SAR processing scheme, which is much more robust concerning long synthetic apertures and squint angle than for example the chirp-scaling method. Additionally it offers the possibility of processing wideband low-frequency airborne SAR data up to near-wavelength resolution.**

The performance and the accuracy of the new EOK SAR data processing algorithm is demonstrated using simulated data.

I. Introduction

A crucial problem in most airborne SAR sensors is the compensation of motion errors, induced by atmospheric turbulence (i.e. the compensation of changes of the platform forward velocity vector in orientation and/or in magnitude). If not corrected, the image quality will considerably degrade. The main effects observed are the loss of geometric resolution and radiometric accuracy, reduction of image contrast, azimuth ambiguities and strong phase distortions. As modern SAR systems are continuously developing into the direction of higher spatial resolution, processing algorithms are required which are able to deal with high bandwidths combined with wide azimuthal integration intervals, even in the presence of motion errors.

In literature, two processing methods have been found to be suited: Wavenumber domain (Omega-k) processing [1] and Extended Chirp Scaling (ECS) [2]. Omega-k processing is commonly accepted to be an ideal solution of the SAR focusing problem as long as straight sensor trajectory is given. For motion error affected data it cannot be applied. Here, the Extended Chirp Scaling (ECS) algorithm is proven to be very powerful, but it has limitations concerning long aperture synthesis and heavily squinted geometries [3]. In the following, a new variant of wavenumber domain processing should is proposed, which is able to cope with motion errors.

An accurate motion-compensation (MoCo) requires to compensate for each target all the line-of-sight displacements and the corresponding phase rotations during azimuth integration, which are caused by the aircraft movement. Resulting from the non-straight trajectory of an aircraft, motion errors are dependent from the azimuth position, and, due to changes in the look-angle from near-range to far-range, also from range distance. In modern SAR processors for focusing airborne SAR data, often a two-step motion compensation approach is performed. In this approach, the motion error is splitted into a range-independent and a range-dependent component. This is made due to the fact that a range-dependent motion compensation can only be performed correctly after range focusing and correction of the range-cell-migration (RCM).

II. Conventional wavenumber processing

Wavenumber domain SAR focusing is based on the so-called radiating reflector model [4]. In this model, all scatterer in the scene are considered to start simultaneously to radiate a spherical wave pulse at time $t = -t_0$. The total wave field $\psi_{tot}(x, t, r = 0)$ measured by the antenna at time t at the azimuth position x and slant range position $r = 0$ is given by the coherent superposition of all the spherical waves emitted at $t = -t_0$. The inverse problem, i.e. the SAR focusing, attempts to propagate the wave field, sampled at the sensor positions at the time t, back in time to $t = -t_0$. The result should correspond directly to the initial wave field; and, therefore, to the distribution of the scatterers.

As described in detail in [1], the focusing equation of the wavenumber algorithm can be expressed as

$$\psi_{tot}(x, -t_0, r) = \frac{1}{(2\pi)^2} \iint \tilde{\Psi}_{tot}(k_x, \omega, r = 0) \cdot \exp\left[i \left(k_x x - \omega t_0 + \frac{2\omega r}{c} \sqrt{1 - \frac{k_x^2 c^2}{4\omega^2}} \right) \right] \mathrm{d}\omega \, \mathrm{d}k_x \tag{1}$$

with $\tilde{\Psi}_{tot}(k_x, \omega, r = 0$ denoting the complex amplitude of a spherical wave with wavenumber in azimuth k_x and frequency ω, recorded at the sensor trajectory ($r = 0$). It has a parametric dependency on the range distance r, making its solution computationally very inefficient. Therefore, Eq. 1 is usually transformed into the form of a two-dimensional FOURIER-integral by substituting $\omega = \frac{c}{2}\sqrt{k_x^2 + k_r^2}$. This step, the so-called "STOLT-mapping", represents among other things an interpolation of the data spectrum, which maps lines of constant ω into circles with radius $2\omega/c$ in the (k_x, k_r)-domain. Together with an appropriate phase multiplication, this corrects the hyperbolic range-cell-migration (RCM) and focuses the image.

In conventional wavenumber processing, the inclusion of a range-dependent motion compensation step is not possible, as for this it is required that the RCM is corrected, while the data are still unfocused in azimuth.

III. Extended Wavenumber Algorithm

Eq. 1 represents the focusing equation of 'ω-k'-processing. As described before, it is usually transformed into the form of a two-dimensional FOURIER-integral. An alternative way of transforming Eq. 1 into a different FOURIER-integral, which separates the RCM correction from the actual azimuthal focusing, will be described in the following.

By inserting a zero-term, Eq. 1 can be expanded as in the following:

$$\begin{aligned}\psi_{tot}(x,-t_0,r) = \\ &\frac{1}{(2\pi)^2}\iint \tilde{\Psi}_{tot}\left(k_x,\omega,r=0\right)\cdot\exp(i(k_x x-\omega t_0)) \\ &\cdot\exp\left[ir\left(\sqrt{\left(\frac{2\omega}{c}\right)^2-k_x^2}-\sqrt{\left(\frac{2\omega_0}{c}\right)^2-k_x^2}\right)\right] \\ &\cdot\exp\left[ir\sqrt{\left(\frac{2\omega_0}{c}\right)^2-k_x^2}\right]\,\mathrm{d}k_x\,\mathrm{d}k_r\,. \qquad (2)\end{aligned}$$

At this point, a different change-of-variable than for the "STOLT-mapping" is applied:

$$\hat{k}_r = \sqrt{\left(\frac{2\omega}{c}\right)^2-k_x^2}-\sqrt{\left(\frac{2\omega_0}{c}\right)^2-k_x^2} \qquad (3)$$

It represents again an interpolation of the data spectrum. But instead of mapping lines of constant ω into circles with radius $2\omega/c$ in the (k_x, k_r)-domain, the proposed modified "STOLT-mapping" additionally introduces a frequency shift in $\hat{k}_r$ direction. As depicted in Fig. 1, this frequency shift causes all points with frequency ω_0 to stay on a line with constant $\hat{k}_r$. The $\hat{k}_r$ and k_r axis are parallel, but displaced by $\sqrt{\left(\frac{2\omega_0}{c}\right)^2-k_x^2}$.

Applying the proposed change-of-variable operation to Eq. 2, the following expression is derived:

$$\begin{aligned}\psi_{tot}(x,-t_0,r) = &\frac{1}{(2\pi)^2}\int \exp\left[ir\sqrt{\left(\tfrac{2\omega_0}{c}\right)^2-k_x^2}\,\right] \\ &\cdot\int\tilde{\Psi}_{tot}\left(k_x,\frac{c}{2}\sqrt{\left(\hat{k}_r+\sqrt{\left(\tfrac{2\omega_0}{c}\right)^2-k_x^2}\right)+k_x^2},r=0\right) \\ &\cdot\exp\left[\frac{ict_0}{2}\sqrt{\left(\hat{k}_r+\sqrt{\left(\tfrac{2\omega_0}{c}\right)^2-k_x^2}\right)+k_x^2}\,\right] \\ &\cdot\frac{c}{2}\frac{\hat{k}_r+\sqrt{\left(\frac{2\omega_0}{c}\right)^2-k_x^2}}{\sqrt{\left(\hat{k}_r+\sqrt{\left(\frac{2\omega_0}{c}\right)^2-k_x^2}\right)+k_x^2}}\cdot e^{i(\hat{k}_r r+k_x x)}\,\mathrm{d}\hat{k}_r\,\mathrm{d}k_x \qquad (4)\end{aligned}$$

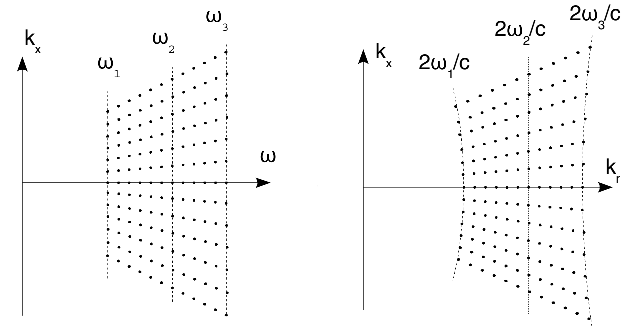


Fig. 1. Modified STOLT-mapping. Left: spectral representation of the acquired raw-data. Right: spectrum transformed to the (k_r, k_x)-domain, with compensation for ω_0.

The first exponential term in Eq. 4 is only dependent from r and the azimuth wavenumber, and not anymore from the range frequency ω. This term is responsible for the final azimuth focusing, after RCM and the frequency dependence of the focusing function is corrected. These two corrections are performed by the second integral in Eq.4. Eliminating the azimuth focusing term, one gets an image which is RCM corrected and where the extensions of the target responses in azimuth correspond exactly to their extensions in the raw data. At this point a range-dependent, second-order motion compensation can be applied. After correcting the range-dependent part of the motion error, the image has to be transformed back in azimuth wavenumber domain by a one-dimensional FFT. There, a phase correction of the form

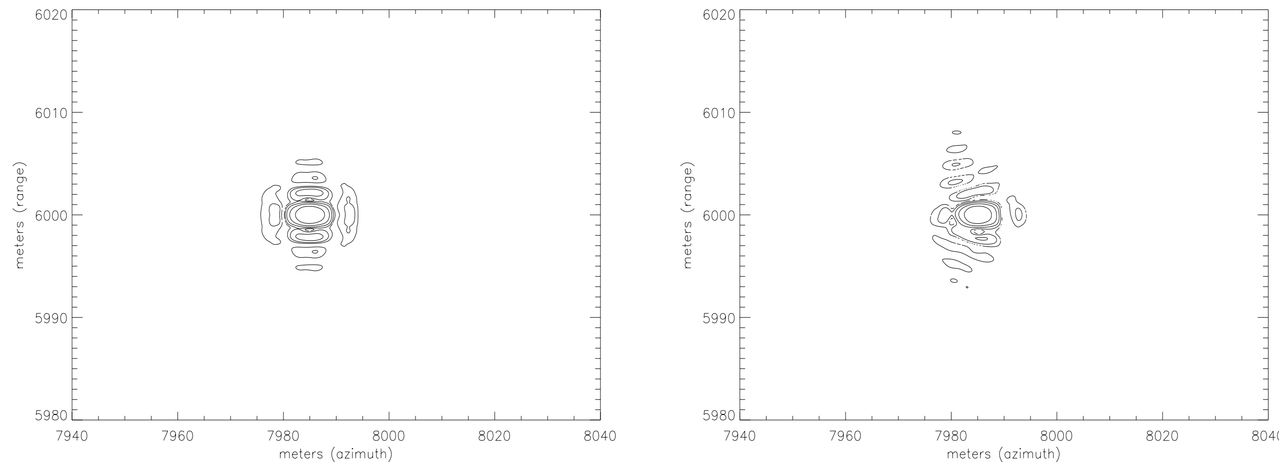
$$\Phi(r,k_x) = \exp\left[ir\sqrt{\left(\frac{2\omega_0}{c}\right)^2-k_x^2}\right] \qquad (5)$$

performs the final azimuth focusing. Finally, an inverse FFT along azimuth has to be applied to transform back the image to the spatial domain in azimuth.

During the motion compensation the data have to be corrected by a phase term proportional to the deviations $\Delta r(x,r)$ between the actual trajectory and a nominal reference track. These deviations are usually considered to be dependent only from azimuth position and range distance. However, in case of a wide angular characteristic of the antenna in azimuth, a different approach has to be taken, because the actual squint-angle under which a target is seen varies strongly during the azimuthal integration time. In [5], an advanced range adaptive sub-aperture motion compensation algorithm has been proposed. It can be used together with the proposed extended wavenumber algorithm to compensate for motion error variations during the azimuth integration time, occuring in case of wide beam antennas.

IV. Experimental results

A low frequency airborne SAR raw data simulation has been performed to evaluate the performance of proposed algorithms in processing low frequency, high along-track resolution, wide-band and wide-beam SAR data. The main simulation parameters are: Wavelength: 2.5m, chirp bandwidth: 30MHz, azimuth



(a) (b)

Fig. 2. Simulated 2D impulse responses of the proposed algorithm. (a) ideal response (no motion errors), (b) with motion errors up to 10m

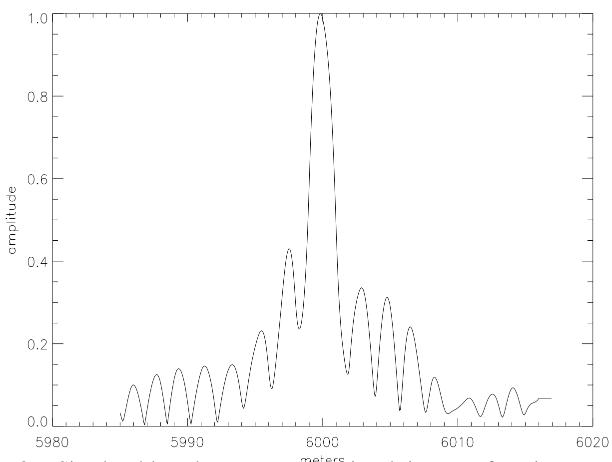


Fig. 3. Simulated impulse response in azimuth in case of motion errors up to 10m at 2.5m wavelength and 50° processed aperture.

antenna aperture: 50°, forward velocity: 100m/s, PRF: 100Hz. In Fig. 2(a) the ideal 2D impulse response (i.e. without any motion errors) of the algorithm is shown.

Additionally, raw data strongly affected by motion errors have been simulated as well. The maximum simulated deviation from ideal (linear) flight track during data collection was about 10m, with a typical wavelength of the motion errors of about 300m. These parameters are typical for a aircraft flying at low flight levels (10000ft). The 2D impulse response obtained from such raw data, using EOK processing combined with azimuthal sub-aperture motion-compensation, is shown in Fig. 2(b). In Fig. 3 the response in azimuth is depicted.

It can be observed that the motion errors degrade the processing quality. The peak-sidelobe ratio (PSLR) increases from 10.5dB to 7.8dB, the resolution from 1.8m to 2.0m. These values can be considered to be very good, taking into account that no wavelength dependent motion compensation has been performed.

V. Conclusions

In contrast to conventional 'ω-k' processing, the new EOK algorithm can directly integrate a two-step motion-compensation in the general formulation of the wavenumber domain algorithm. This concept can be completed by the use range adaptive sub-aperture algorithm for residual MoCo error correction. Using simulated data, it was proven that high image quality can be achieved using this approach.

A disadvantage of the proposed algorithm is that generally motion compensation has to be considered to be wavelength dependent. Therefore, for systems with very high relative bandwidths and wide antenna apertures, sub-apertures in range and azimuth are required to achieve high image quality. This could possibly make wavenumber domain processing inefficient. However, the presented simulation results that for not too extreme scenarios the algorithm is very powerful.

The proposed algorithms seems to be very promising for processing highly squinted data, as the general formulation of the wavenumber algorithm is not bound to low squint angles. An implementation of the algorithm for processing strongly squinted data to very high-resolutions is currently under development.

References

[1] C. Cafforio, C. Prati, and F. Rocca: "SAR Data Focusing Using Seismic Migration Techniques", *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 27, No. 2, pp. 194-207, 1991.

[2] A. Moreira, J. Mittermayer and R. Scheiber: "Extended Chirp Scaling Algorithm for Air- and Spaceborne SAR Data Processing in Stripmap and ScanSAR Imaging Modes", *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 34, No. 5, pp. 1123-1136, Sep. 1996.

[3] R. Bamler: "A comparison of range-doppler and wavenumber domain SAR focusing algorithms", *IEEE Transactions n Geoscience and Remote Sensing*, Vol. 30, pp. 706-713, July 1992.

[4] R. Stolt: "Migration by Fourier Transform Techniques", *Geophysics*, Vol. 43, No. 1, pp. 49-76, 1978.

[5] A. Potsis, A. Reigber, J. Mittermayer, A. Moreira and N. Uzunoglou: "Sub-aperture algorithm for motion compensation improvement in wide-beam SAR data processing". *Electronic Letters*, Vol. 37, No. 23, pp. 1405-1407, 2001.